CHAPTER 3

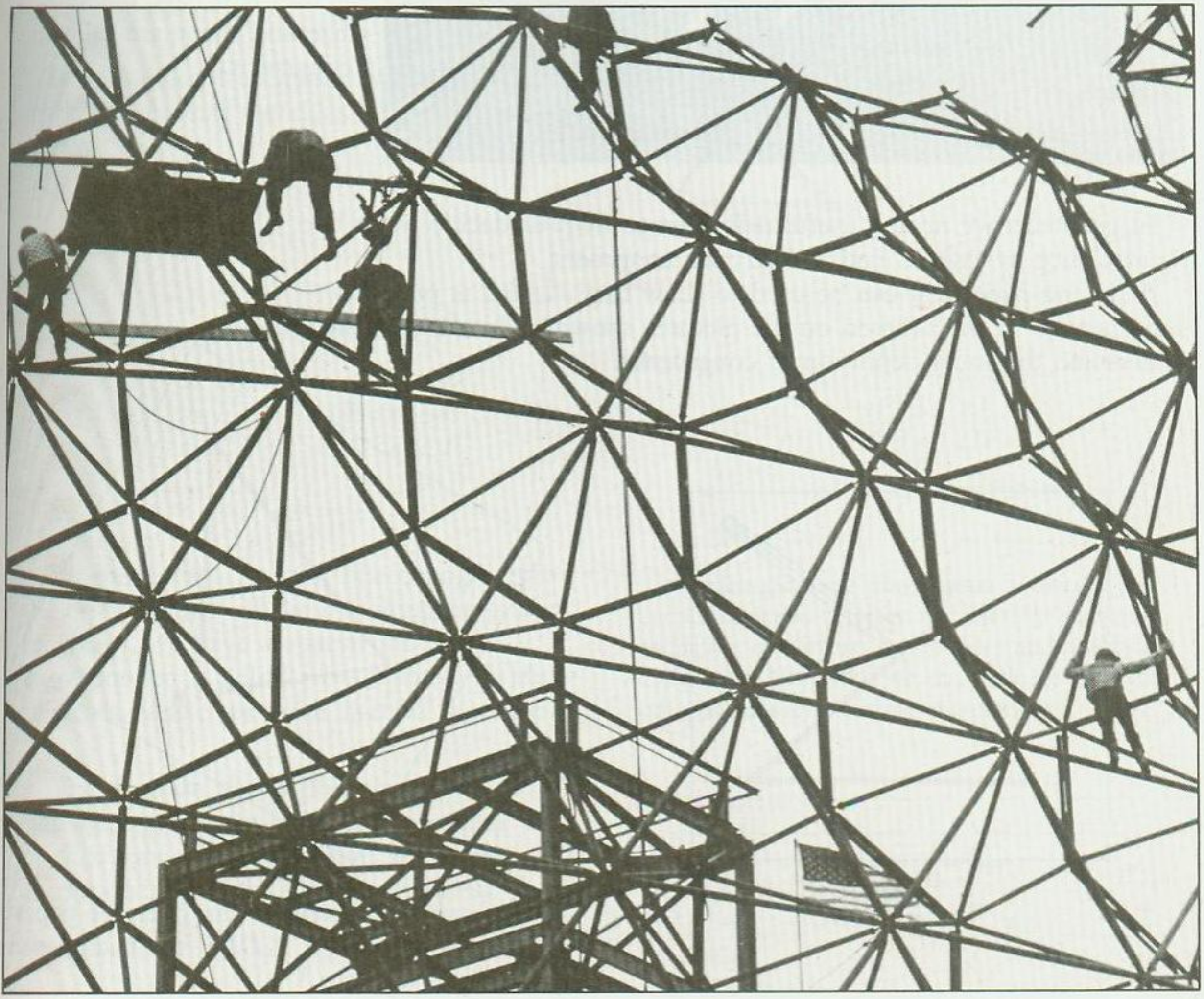
3–1 Congruent Triangles 84
3–2 Congruence Postulates 90
3–3 Proofs: Using the Congruence Postulates 96
3–4 Proofs: Using Definitions 100
3–5 Proofs: Using Postulates and Definitions 104
3–6 Proving Segments and Angles Congruent 110
3–7 Proofs: Overlapping Triangles 116
3–8 Proofs: Chains of Congruences 120

Important Ideas 122 *Review* 123 *Test* 124

Cumulative Review (Chapters 1–3) 125

Geometry In Our World
Architecture: Geodesic Domes 126

Triangles and Congruence

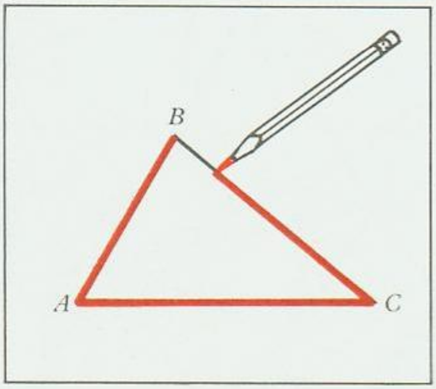


3–1 Congruent Triangles

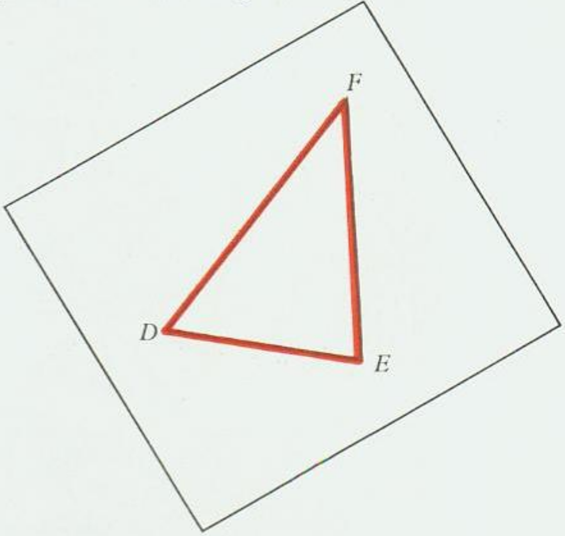
Automobiles are manufactured using assembly line production. The parts that are produced must be identical in size and shape so that they can be used in any car on the assembly line. Replacement parts must also be identical. In geometry, figures that have the same size and shape are called congruent.

In geometry we need a useful definition to help us decide when two figures, such as $\triangle ABC$ and $\triangle DEF$, are congruent.

A tracing paper test can be used to show that $\triangle ABC$ is probably congruent to $\triangle DEF$ (based on the pictures shown). This test can help us understand the usual definition of congruence.

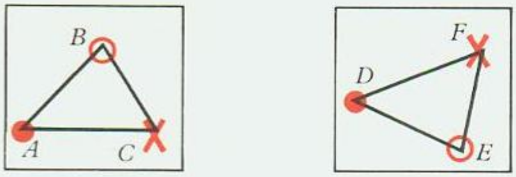


Trace $\triangle ABC$.



If you can slide, turn, or flip over the paper so the tracing fits on $\triangle DEF$, the triangles are probably congruent.

Consider the tracing test again. If we highlight each vertex with a special symbol we see that the vertices of the two triangles correspond as shown. The angle correspondence is also determined.



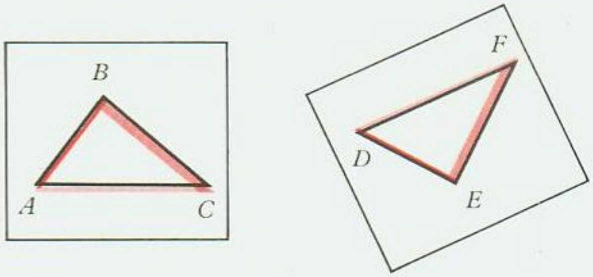
$A \leftrightarrow D$
$B \leftrightarrow E$
$C \leftrightarrow F$

vertex correspondence

$\angle A \leftrightarrow \angle D$
$\angle B \leftrightarrow \angle E$
$\angle C \leftrightarrow \angle F$

angle correspondence

If we highlight each side with a special mark, we see that the sides correspond as shown.



In this tracing test we observe that

$\overline{AB} \cong \overline{DE}$ $\quad \angle A \cong \angle D$
$\overline{BC} \cong \overline{EF}$ $\quad \angle B \cong \angle E$
$\overline{AC} \cong \overline{DF}$ $\quad \angle C \cong \angle F$

$\overline{AB} \leftrightarrow \overline{DE}$
$\overline{BC} \leftrightarrow \overline{EF}$
$\overline{AC} \leftrightarrow \overline{DF}$

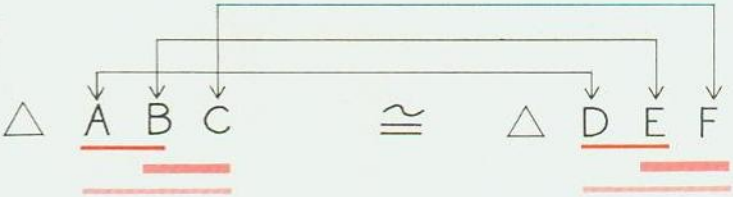
side correspondence

Whenever we can "fit" one triangle onto the other so that all the matching parts are *congruent,* we have a special kind of correspondence called a *congruence.* To show this congruence we write $\triangle ABC \cong \triangle DEF$.

Definition 3–1

Two **triangles are congruent** if there is a correspondence between the vertices such that each pair of corresponding sides and angles are congruent. Note that we can define congruence for other figures in a similar manner.

The diagram shows how this statement about the congruent triangles provides specific information about the corresponding parts (angles and sides).

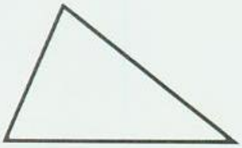


EXERCISES

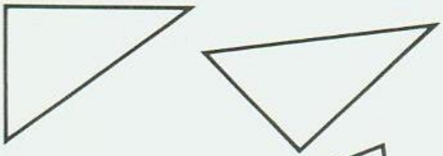
A.

Which of the following pairs of figures are congruent? (You may want to use tracing paper.)

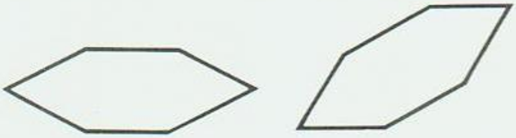
1.



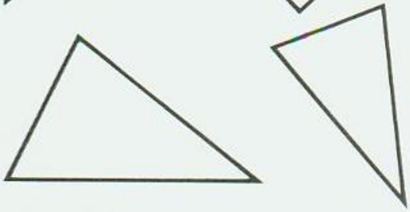
2.



3.

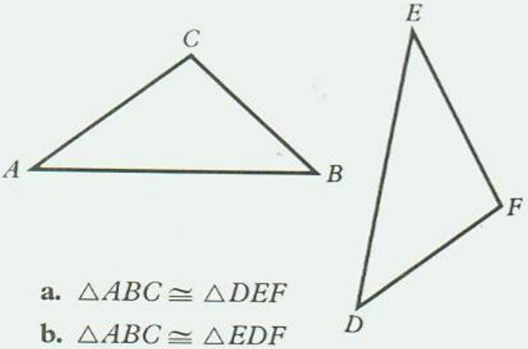


4.



In exercises 5–8 select the correct statement. (Use tracing paper if needed.)

5.

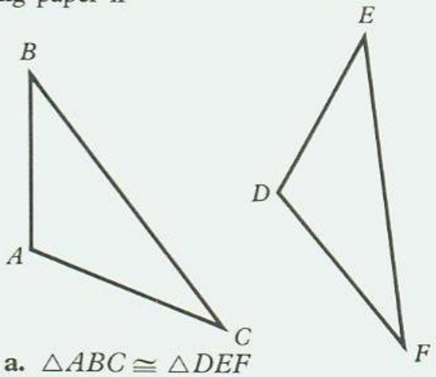


a. $\triangle ABC \cong \triangle DEF$

b. $\triangle ABC \cong \triangle EDF$

c. $\triangle ABC \cong \triangle EFD$

6.

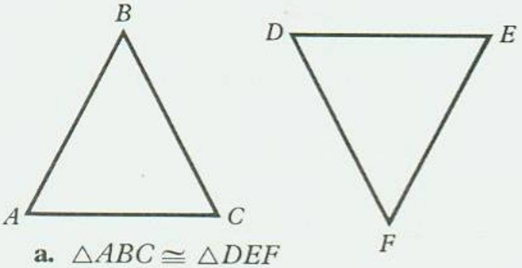


a. $\triangle ABC \cong \triangle DEF$

b. $\triangle ABC \cong \triangle DFE$

c. $\triangle ABC \cong \triangle FED$

7.

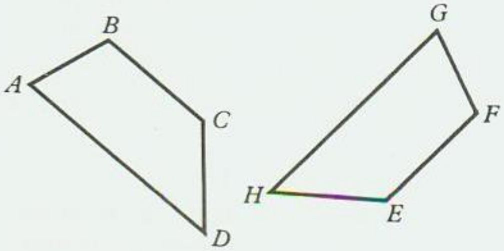


a. $\triangle ABC \cong \triangle DEF$

b. $\triangle ABC \cong \triangle EFD$

c. $\triangle ABC \cong \triangle FDE$

8.



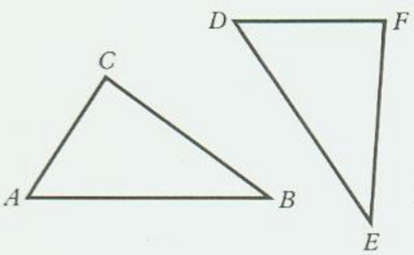
a. $ABCD \cong EFGH$

b. $ABCD \cong FGHE$

c. $ABCD \cong GHEF$

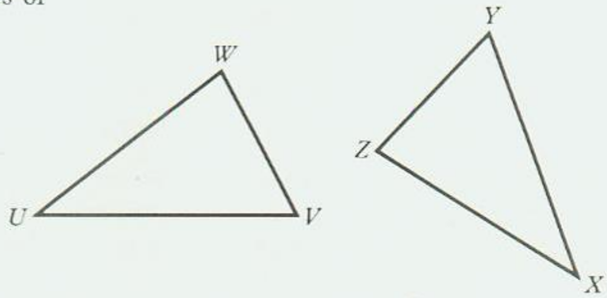
d. $ABCD \cong GFEH$

9. Given that $\triangle ABC \cong \triangle DEF$, select the false statement in each part.
   - a. $\overline{AC} \cong \overline{DF}$, $\angle B \cong \angle E$, $\overline{BC} \cong \overline{DE}$, $\angle C \cong \angle F$
   - b. $\overline{AB} \cong \overline{ED}$, $\angle A \cong \angle D$, $\angle C \cong \angle F$, $\overline{AB} \cong \overline{EF}$
   - c. $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{FE}$, $\angle C \cong \angle D$, $\overline{AC} \cong \overline{DF}$



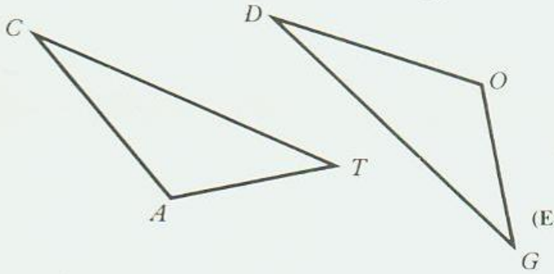
(Ex. 9)

10. Given that $\triangle UVW \cong \triangle XYZ$, complete the congruences for the six pairs of corresponding congruent parts.
   - a. $\angle U \cong$ ______?______
   - b. $\angle V \cong$ ______?______
   - c. $\angle W \cong$ ______?______
   - d. $\overline{UV} \cong$ ______?______
   - e. $\overline{UW} \cong$ ______?______
   - f. $\overline{VW} \cong$ ______?______



(Ex. 10)

11. Given that $\triangle CAT \cong \triangle DOG$, write the congruences for the six pairs of corresponding congruent parts.

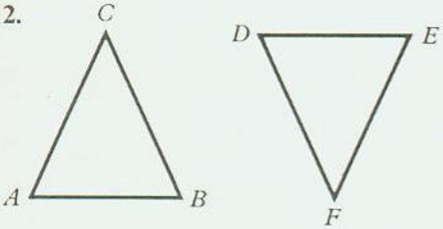


(Ex. 11)

B.

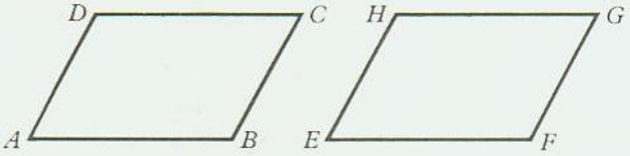
In each of exercises 12 and 13 there are two correct congruence statements. Find them.

12.



- a. $\triangle ABC \cong \triangle DEF$
- b. $\triangle ABC \cong \triangle EFD$
- c. $\triangle ABC \cong \triangle FDE$
- d. $\triangle ABC \cong \triangle EDF$

13.



- a. $ABCD \cong EFGH$
- b. $ABCD \cong FGHE$
- c. $ABCD \cong GHEF$
- d. $ABCD \cong FEHG$

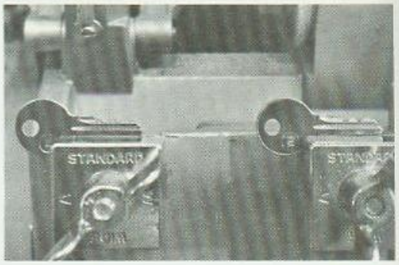
14. Given that $\triangle PRS \cong \triangle JKL$ write three congruence statements about angles and three congruence statements about sides of the triangles.

15. Given these six congruence statements, complete correctly the statement about congruent triangles.

$\angle A \cong \angle B$ $\quad \overline{AP} \cong \overline{BT}$
$\angle T \cong \angle P$ $\quad \overline{AR} \cong \overline{BJ}$
$\angle R \cong \angle J$ $\quad \overline{PR} \cong \overline{TJ}$
$\triangle \underline{?}\ \underline{?}\ \underline{?} \cong \triangle \underline{?}\ \underline{?}\ \underline{?}$

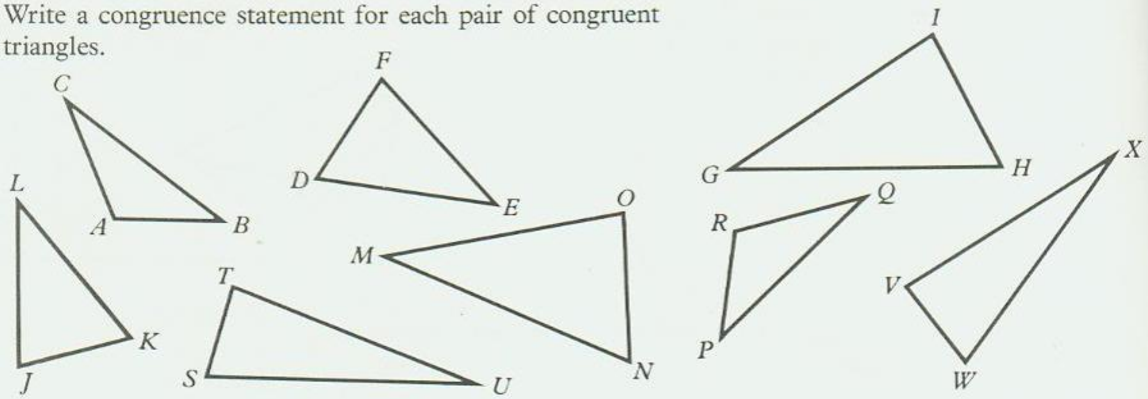
16. Given that $\triangle ABC \cong \triangle DEF$, two of the following congruence statements between triangles are also true. Which two?

a. $\triangle BCA \cong \triangle EFD$ **b.** $\triangle ACB \cong \triangle EFD$
c. $\triangle CBA \cong \triangle FDE$ **d.** $\triangle CAB \cong \triangle FDE$



17. When a locksmith cuts a new key three arbitrary points A, B, and C on the surface of the given key correspond to three points A', B', and C' on the new key. How does $\triangle ABC$ relate to $\triangle A'B'C'$?

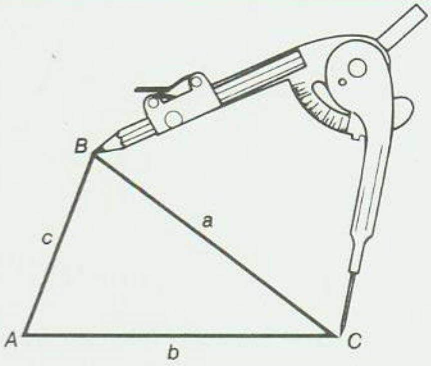
18. Write a congruence statement for each pair of congruent triangles.



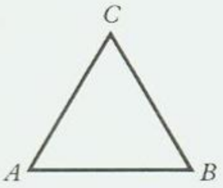
Activity

Every triangle, like $\triangle ABC$, has six parts. There are three sides (a, b, and c) and three angles ($\angle A$, $\angle B$, and $\angle C$).

1. Suppose you are allowed only to open your compass to the lengths of the three sides when needed. Construct a triangle congruent to $\triangle ABC$.
2. Suppose you are allowed only to open your compass to the length of side c, side b, and to copy $\angle A$. Construct a triangle congruent to $\triangle ABC$.
3. How many other combinations of three of the six parts will allow you to construct a triangle congruent to $\triangle ABC$?



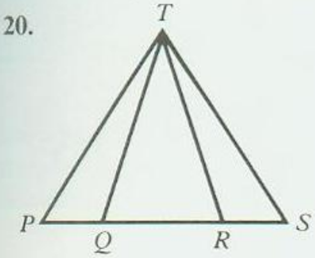
19. If $\triangle ABC$ is an equilateral triangle, we can write $\triangle ABC \cong \triangle BCA$. There are five more congruence statements that can be written. Write them.



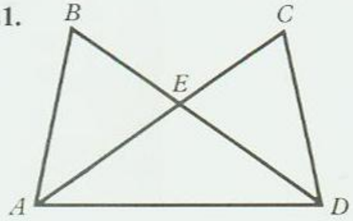
C.

Each figure below contains one or more pairs of congruent triangles. Sometimes the triangles "overlap" each other. Write a correct congruence statement for each pair you find in a figure.

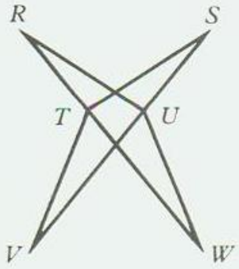
20.



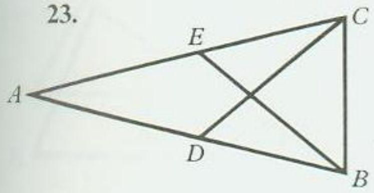
21.



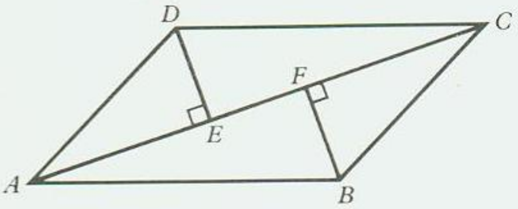
22.



23.



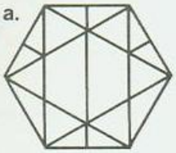
24.



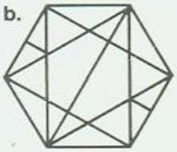
PROBLEM SOLVING

1. Which pairs of figures are identical?

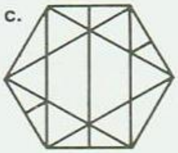
a.



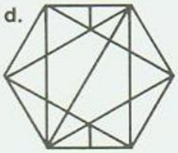
b.



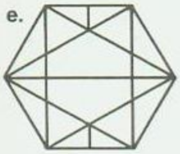
c.



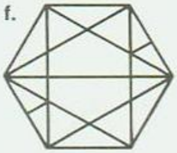
d.



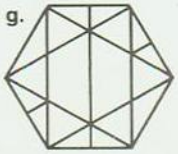
e.



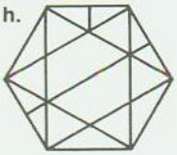
f.



g.



h.

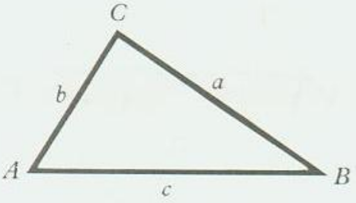


2. Trace figure a and label points where segments intersect. Can you find six pairs of congruent triangles (each involving different sized or shaped triangles) and write the correct congruence statements?

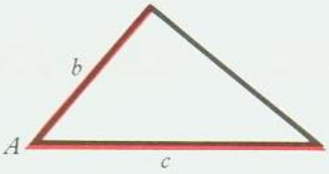
3–2 Congruence Postulates

Suppose a carpet layer needs to repair a carpet with a triangular patch. It is not likely that all three sides and all three angles would be measured in order to cut out the patch. Only a few of these measurements are needed. In this lesson we shall discuss how many and which combinations of the six parts of a triangle (sides a, b, c and angles A, B, C) are needed to determine a particular size and shape of a triangle.

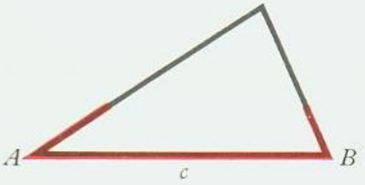
First, we state some ideas that will be useful in our work with congruent triangles.



An angle and the side opposite the angle are labeled with the same letter—the angle with a capital letter, and the side with a small letter.

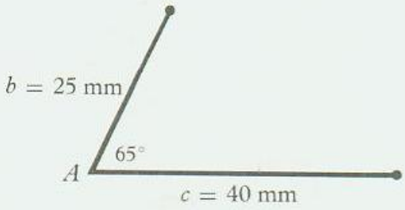


Two sides include an angle if the vertex of the angle is an endpoint of both sides.

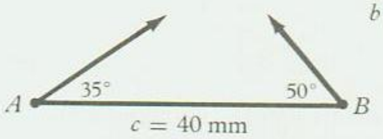


Two angles include a side if the endpoints of the side are vertices of the two angles.

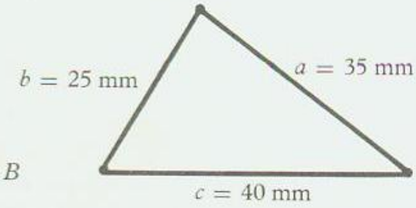
If you construct a triangle given three of the six parts, you find that the size and shape of the triangle is completely determined when you are given the following information.



1. Two sides and an included angle.

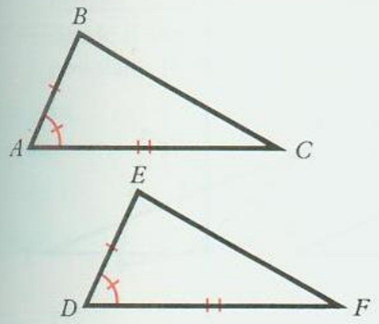


2. Two angles and the included side.



3. Three sides.

From this we obtain the following three congruence postulates.

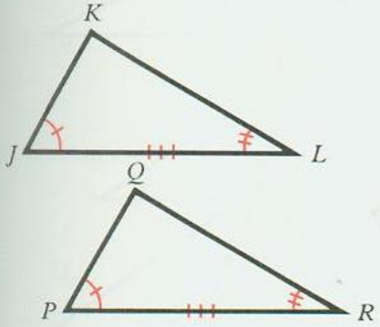


The marks on the triangles indicate which sides and angles are congruent.

We believe that only one triangle can result if two sides and the included angle are given, so we accept this postulate.

SAS Congruence Postulate

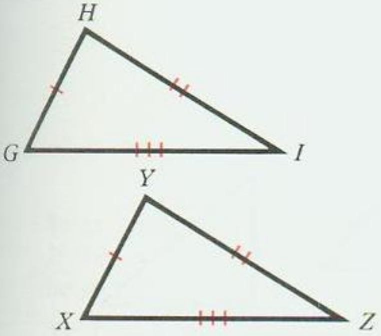
If two sides and the included angle of one triangle are congruent respectively to two sides and the included angle of another triangle, then the two triangles are congruent.



We believe that only one triangle can result if two angles and the included side are given, so we accept this postulate.

ASA Congruence Postulate

If two angles and the included side of one triangle are congruent respectively to two angles and the included side of another triangle, then the two triangles are congruent.



We believe that only one triangle can result if three sides are given, so we accept this postulate.

SSS Congruence Postulate

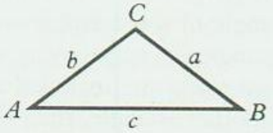
If all three sides of one triangle are congruent respectively to all three sides of another triangle, then the two triangles are congruent.

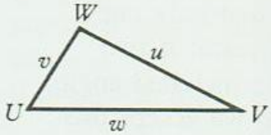
The definition of congruent triangles shows six pairs of parts should match. But these three postulates suggest that sometimes we need to check only three of these pairs to be sure we have a congruence. We will use these postulates to prove additional theorems about triangles.

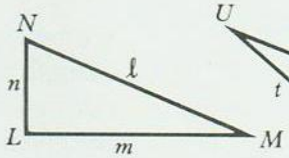
EXERCISES

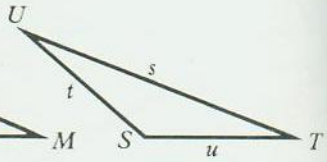
A.

1. Which of these triangles is not labeled correctly?

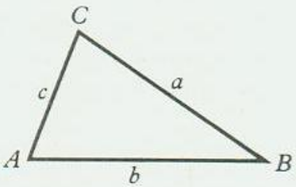


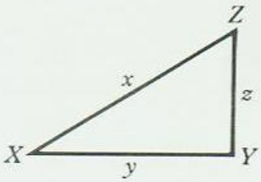


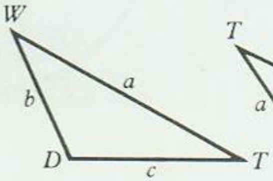


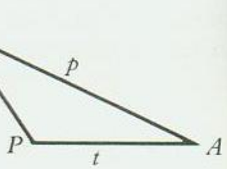


2. Which of these triangles is labeled correctly?





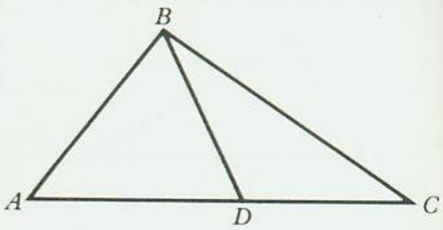




3. Draw a triangle and label its angles and sides using the standard method of labeling.

For each pair of sides (or angles) state the included angle (or side).

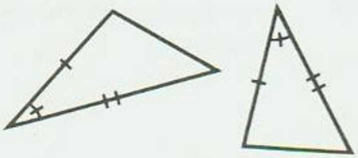
4. $\overline{AB}$ and $\overline{AD}$
5. $\overline{BD}$ and $\overline{BC}$
6. $\angle A$ and $\angle C$
7. $\angle ABD$ and $\angle ADB$
8. $\overline{AD}$ and $\overline{BD}$
9. $\overline{BD}$ and $\overline{CD}$
10. $\angle ABC$ and $\angle C$



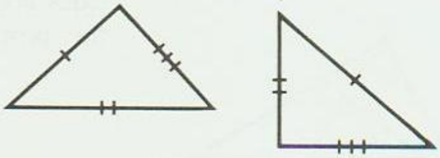
(Exs. 4-10)

By which of the three congruence postulates (SAS, ASA, SSS) are these pairs of triangles congruent? (Assume the congruences are as the markings indicate even though the triangles may not look congruent.)

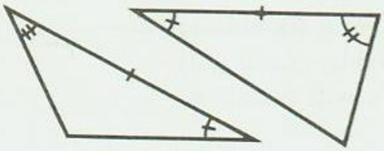
11.



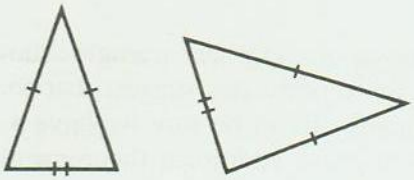
12.



13.

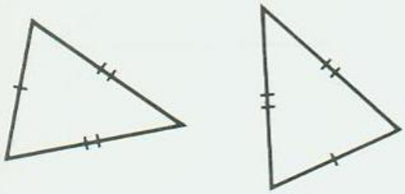


14.

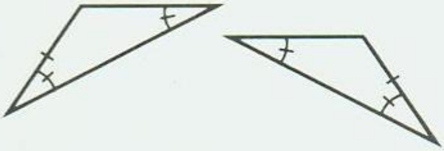


In exercises 15–22 the congruent parts of the triangles are marked. State whether or not there is enough information to decide if the triangles are congruent by the SAS, ASA, or SSS postulate. If they are congruent, state the postulate.

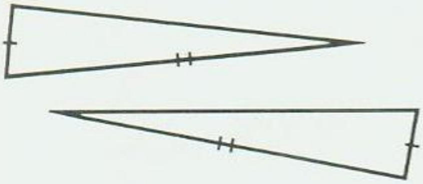
15.



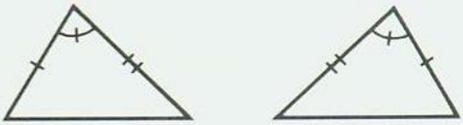
16.



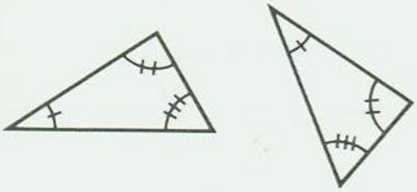
17.



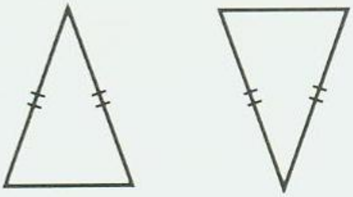
18.



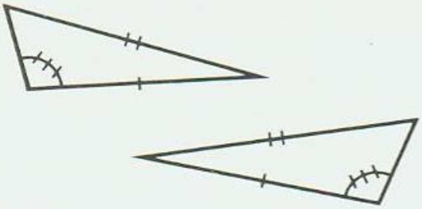
19.



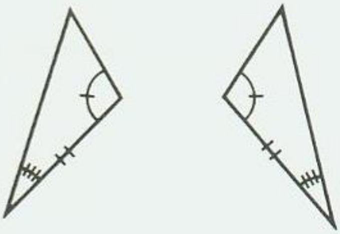
20.



21.



22.



B.

In exercises 23–27 tell whether or not the triangles are congruent. If they are congruent, tell which postulate (SAS, ASA, SSS) can be used to verify this.

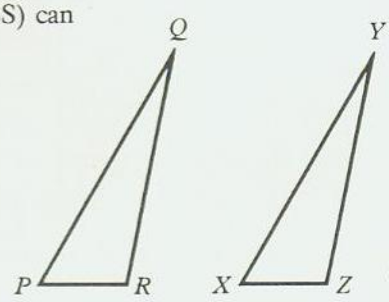
23. **Given:** $\overline{PQ} \cong \overline{XY}$, $\overline{QR} \cong \overline{YZ}$, $\overline{PR} \cong \overline{XZ}$.

24. **Given:** $\overline{PR} \cong \overline{XZ}$, $\overline{RQ} \cong \overline{ZY}$, $\angle R \cong \angle Z$.

25. **Given:** $\angle P \cong \angle X$, $\angle R \cong \angle Z$, $\overline{PQ} \cong \overline{XY}$.

26. **Given:** $\angle Q \cong \angle Y$, $\angle R \cong \angle Z$, $\overline{QR} \cong \overline{YZ}$.

27. **Given:** $\angle P \cong \angle X$, $\angle Q \cong \angle Y$, $\angle R \cong \angle Z$.

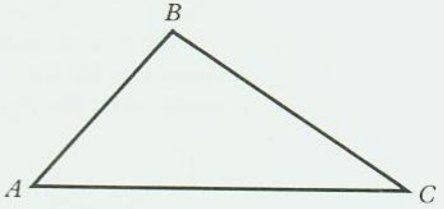


(Exs. 23–27)

28. Use the idea of the SSS postulate to construct a triangle congruent to $\triangle ABC$.

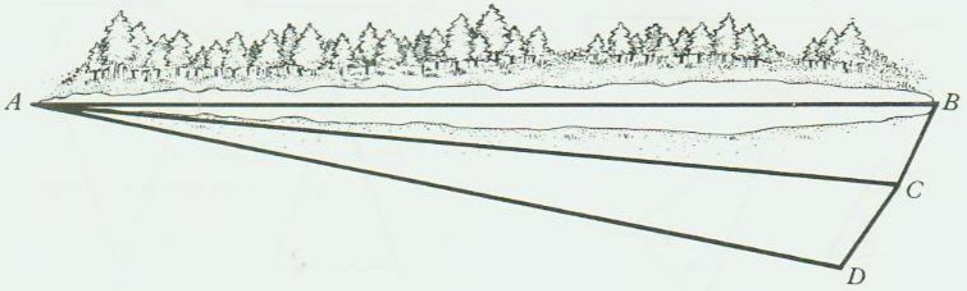
29. Use the idea of the SAS postulate to construct a triangle congruent to $\triangle ABC$.

30. Use the idea of the ASA postulate to construct a triangle congruent to $\triangle ABC$.



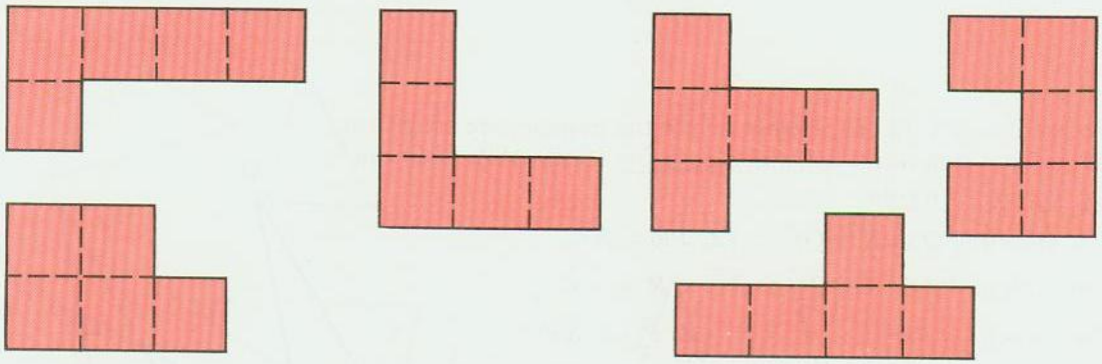
(Exs. 28-30)

31. A team of surveyors want to find the distance AB across a lake. One method requires the construction of a pair of congruent triangles. They select any point C, measure $\angle ACB$, and locate a point D so that $\angle ACD \cong \angle ACB$ and $\overline{CD} \cong \overline{CB}$. Why are $\triangle ACD$ and $\triangle ACB$ congruent? How does this help find the distance across the lake?



Activity

Make cardboard copies (at least three times the size) of the six pentominoes shown. (A pentominoe is a polygon which bounds five squares on a checker board.)

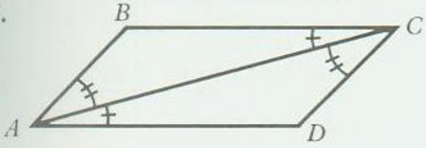


Arrange them to form a pair of rectangles identical in size and shape.

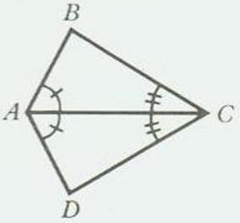
C.

In exercises 32–37 write a congruence statement for each pair of congruent triangles. Also state whether the triangles are congruent by ASA, SAS, or SSS. (Note: Use the idea that a segment is congruent to itself and an angle is congruent to itself.)

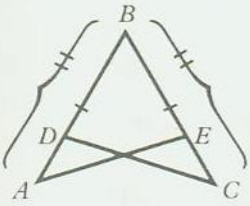
32.



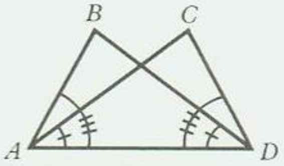
33.



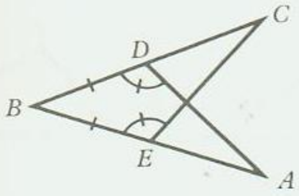
34.



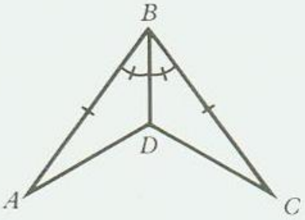
35.



36.

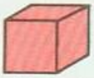


37.

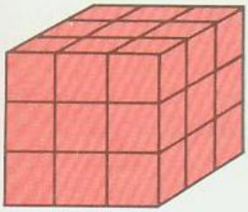


PROBLEM SOLVING

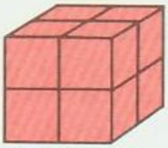
How many ways can a 3 × 3 × 3 cube be positioned in a cube of dimension 4 × 4 × 4? How many 2 × 2 × 2 or 1 × 1 × 1 positions are possible? Complete the table.



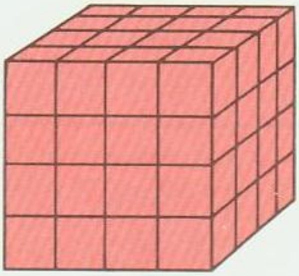
1 × 1 × 1 cube



3 × 3 × 3 cube



2 × 2 × 2 cube



cube size	number of positions in a 4 × 4 × 4 cube

3–3 Proofs: Using the Congruence Postulates

To prove two triangles congruent, we begin with given information and use *patterns of deductive reasoning* to conclude that they are indeed congruent. Affirming the hypothesis is the pattern most often used, as described below.

REVIEW: Affirming the hypothesis is a pattern of reasoning represented as follows:

Whenever $p \rightarrow q$ is true
and p is affirmed to be true,

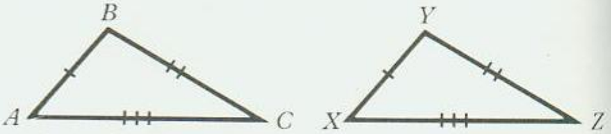
---

we can conclude: q is true.

The SSS Postulate has the general form:

If	all three sides of one triangle are congruent respectively to all three sides of another triangle,	then	the two triangles are congruent.
	p		q.

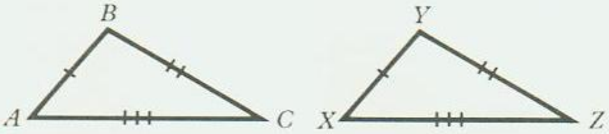
Now we look at a specific application of this statement:



We observe that all conditions stated in p are satisfied, that is, p is true. We have *affirmed the hypothesis*. Therefore we can conclude: q is true. Applying this to our specific case, $\triangle ABC \cong \triangle XYZ$.

It is helpful to organize our thinking by writing the above *proof* in two columns. The left column is used for statements that lead to the desired conclusion. The right column gives reasons why the statements are true.

Given: $\overline{AB} \cong \overline{XY}$
$\overline{BC} \cong \overline{YZ}$
$\overline{AC} \cong \overline{XZ}$
Prove: $\triangle ABC \cong \triangle XYZ$

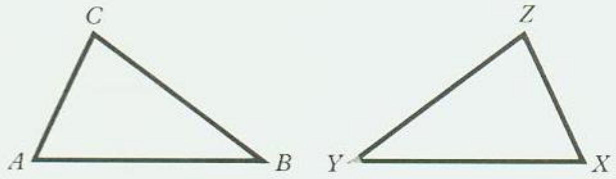


Proof

Statements	Reasons
1. $\overline{AB} \cong \overline{XY}$	1. Given
2. $\overline{BC} \cong \overline{YZ}$	2. Given
3. $\overline{AC} \cong \overline{XZ}$	3. Given
4. $\triangle ABC \cong \triangle XYZ$	4. SSS Congruence Postulate

Here are two more examples of simple proofs that two triangles are congruent. Notice the use of affirming the hypothesis.

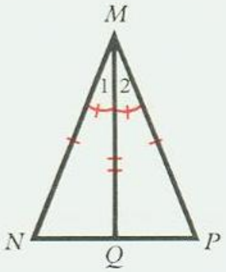
Example 1 **Given:** $\overline{AB} \cong \overline{XY}$
$\angle A \cong \angle X$
$\angle B \cong \angle Y$
Prove: $\triangle ABC \cong \triangle XYZ$



Proof

Statements	Reasons
1. $\overline{AB} \cong \overline{XY}$	1. Given
2. $\angle A \cong \angle X$	2. Given
3. $\angle B \cong \angle Y$	3. Given
4. $\triangle ABC \cong \triangle XYZ$	4. ASA Congruence Postulate

Example 2 **Given:** $\overline{MN} \cong \overline{MP}$
$\angle 1 \cong \angle 2$
Prove: $\triangle MNQ \cong \triangle MPQ$



Proof

Statements	Reasons
1. $\overline{MN} \cong \overline{MP}$	1. Given
2. $\angle 1 \cong \angle 2$	2. Given
3. $\overline{MQ} \cong \overline{MQ}$	3. A segment is congruent to itself.
4. $\triangle MNQ \cong \triangle MPQ$	4. SAS Congruence Postulate

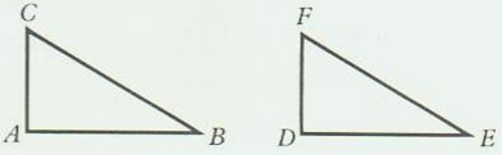
(Note: Sometimes, as in Step 3, a common side of two triangles is involved. The idea that "a segment is congruent to itself" can be used to get a second pair of congruent sides needed to affirm the hypothesis of the SAS congruence postulate. This idea will be discussed further in Chapter 4.)

EXERCISES

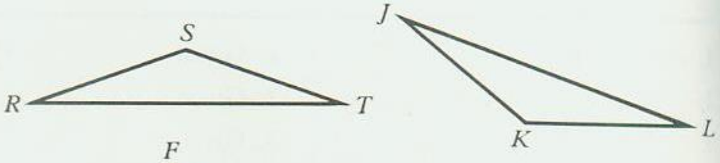
A.

Prove the following triangles congruent by writing a two-column proof. Model your proof after the examples on page 97.

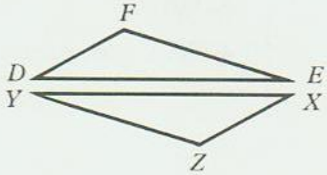
1. **Given:** $\overline{AC} \cong \overline{DF}$
$\overline{AB} \cong \overline{DE}$
$\angle A \cong \angle D$
Prove: $\triangle ABC \cong \triangle DEF$



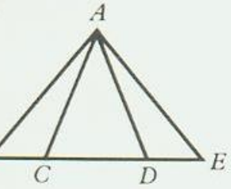
2. **Given:** $\overline{RS} \cong \overline{KL}$
$\overline{ST} \cong \overline{JK}$
$\overline{RT} \cong \overline{JL}$
Prove: $\triangle RST \cong \triangle LKJ$



3. **Given:** $\angle D \cong \angle X$
$\angle F \cong \angle Z$
$\overline{DF} \cong \overline{XZ}$
Prove: $\triangle DEF \cong \triangle XYZ$

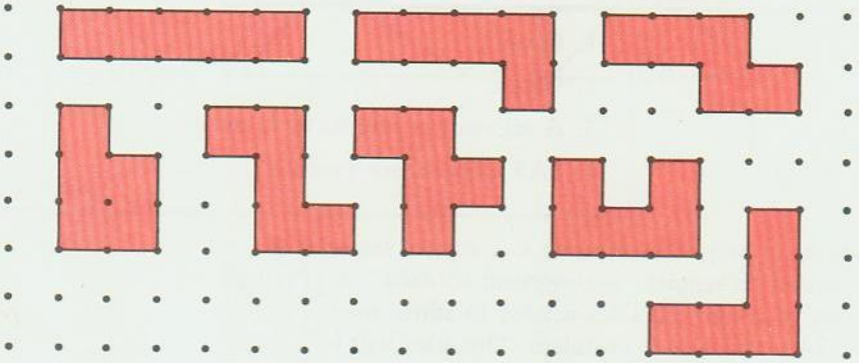


4. **Given:** $\overline{AC} \cong \overline{AD}$
$\overline{AB} \cong \overline{AE}$
$\overline{BC} \cong \overline{DE}$
Prove: $\triangle ABC \cong \triangle AED$

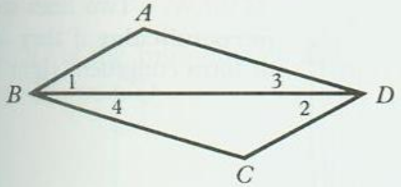


Activity

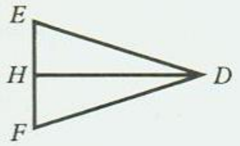
Make large copies of these pentominoes (figures made of five squares) and arrange them into a pair of rectangles identical in size and shape.



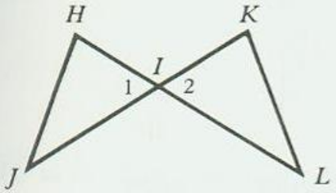
5. **Given:** $\angle 1 \cong \angle 2$
$\angle 3 \cong \angle 4$
Prove: $\triangle ABD \cong \triangle CDB$



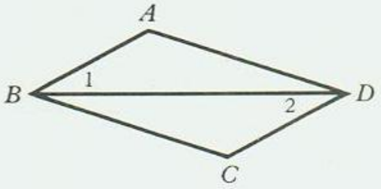
6. **Given:** $\overline{DE} \cong \overline{DF}$
$\overline{EH} \cong \overline{HF}$
Prove: $\triangle DHE \cong \triangle DHF$



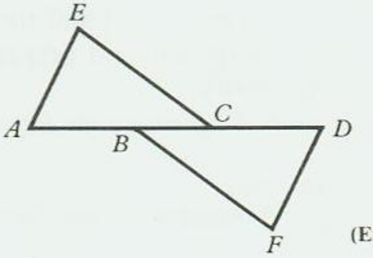
7. **Given:** $\overline{HI} \cong \overline{KI}$
$\angle 1 \cong \angle 2$
$\overline{JI} \cong \overline{IL}$
Prove: $\triangle HIJ \cong \triangle KIL$



8. **Given:** $\overline{AB} \cong \overline{CD}$
$\angle 1 \cong \angle 2$
Prove: $\triangle ABD \cong \triangle CDB$



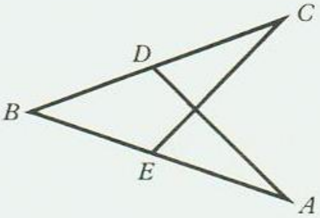
B.



(Exs. 9, 10)

9. **Given:** $\overline{AC} \cong \overline{BD}$
$\angle A \cong \angle D$
$\angle ACE \cong \angle DBF$
Prove: $\triangle ACE \cong \triangle DBF$

10. **Given:** $\overline{AC} \cong \overline{BD}$, $\overline{AE} \cong \overline{DF}$
$\angle A \cong \angle D$
Prove: $\triangle CAE \cong \triangle BDF$



(Exs. 11, 12)

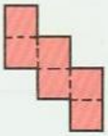
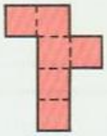
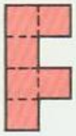
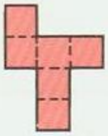
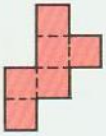
11. **Given:** $\angle A \cong \angle C$, $\overline{AB} \cong \overline{BC}$
Prove: $\triangle CBE \cong \triangle ABD$

12. **Given:** $\angle BDA \cong \angle BEC$
$\overline{BD} \cong \overline{BE}$
Prove: $\triangle BDA \cong \triangle BEC$

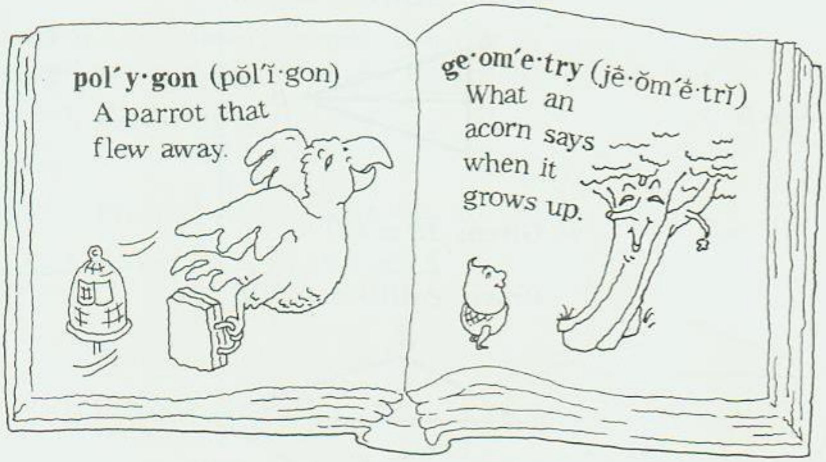
PROBLEM SOLVING

Which of the following patterns do you think can be made into a cube by folding along the edges and taping?

How many other patterns of six squares that will fold to make a cube can you draw?

3–4 Proofs: Using Definitions


pol´y·gon (pŏl´ĭ·gon)
A parrot that flew away.
ge·om´e·try (jē·ŏm´ĕ·trĭ)
What an acorn says when it grows up.

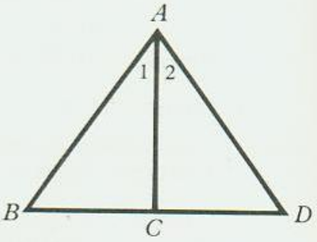
REVIEW: Two lines are **perpendicular** if they intersect to form congruent right angles.

The definition of perpendicular lines (above) and the definitions of angle bisector, midpoint, segment bisector (p. 24), and perpendicular bisector (p. 29) are often used in making proofs.

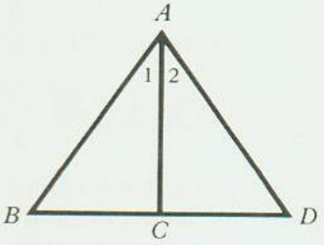
Consider the following example of *affirming the hypothesis* (p. 64).

$p \rightarrow q$: If a ray bisects an angle,
then the two angles formed are congruent.
p: $\overrightarrow{AC}$ bisects $\angle BAD$.

Therefore conclude q: $\angle 1 \cong \angle 2$



The short proof above is shown below using a two-column arrangement.



Given: $\overrightarrow{AC}$ bisects $\angle BAD$
Prove: $\angle 1 \cong \angle 2$

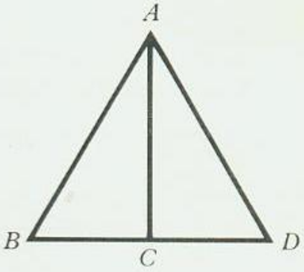
Proof

Statements	Reasons
1. $\overrightarrow{AC}$ bisects $\angle BAD$.	1. Given
2. $\angle 1 \cong \angle 2$	2. Definition of angle bisector

The following short proofs show how the definitions of midpoint, bisector, and perpendicular lines are used. Study each proof.

Example 1

Given: C is the midpoint of $\overline{BD}$.
Prove: $\overline{BC} \cong \overline{CD}$

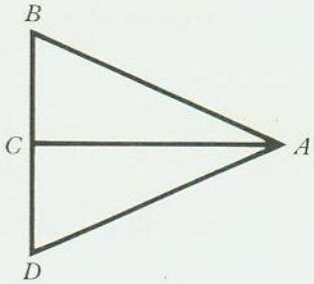


Proof

Statements	Reasons
1. C is the midpoint of $\overline{BD}$.	1. Given
2. $\overline{BC} \cong \overline{CD}$	2. Definition of midpoint

Example 2

Given: $\overline{AC}$ is the perpendicular bisector of $\overline{BD}$.
Prove: C is the midpoint of $\overline{BD}$.

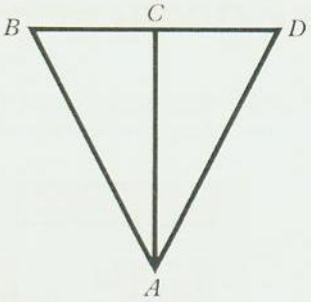


Proof

Statements	Reasons
1. $\overline{AC}$ is the perpendicular bisector of $\overline{BD}$.	1. Given
2. C is the midpoint of $\overline{BD}$.	2. Definition of perpendicular bisector

Example 3

Given: $\overline{AC} \perp \overline{BD}$
Prove: $\angle ACD \cong \angle ACB$



Proof

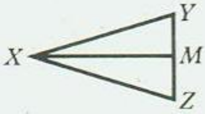
Statements	Reasons
1. $\overline{AC} \perp \overline{BD}$	1. Given
2. $\angle ACD \cong \angle ACB$	2. Definition of perpendicular lines

EXERCISES

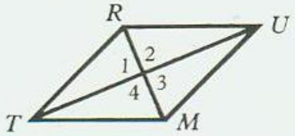
A.

In exercises 1–8 use one of the definitions reviewed in this section to draw a conclusion from the given.

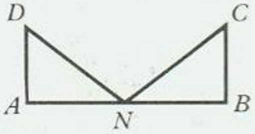
1. **Given:** $\overline{XM}$ bisects $\angle ZXY$.



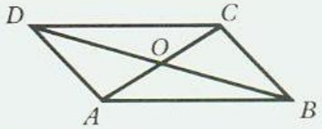
2. **Given:** $\overline{RM} \perp \overline{TU}$



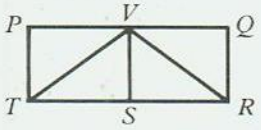
3. **Given:** N is the midpoint of $\overline{AB}$.



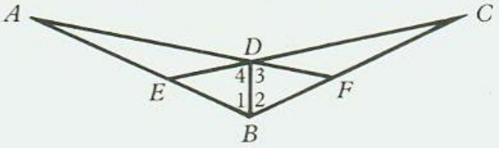
4. **Given:** $\overline{AC}$ and $\overline{BD}$ bisect each other.



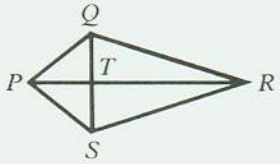
5. **Given:** $\overline{SV}$ is the perpendicular bisector of $\overline{PQ}$.



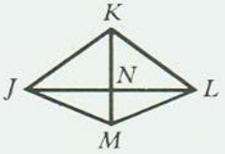
6. **Given:** $\overline{DB}$ bisects $\angle EDF$.



7. **Given:** T is the midpoint of $\overline{QS}$.

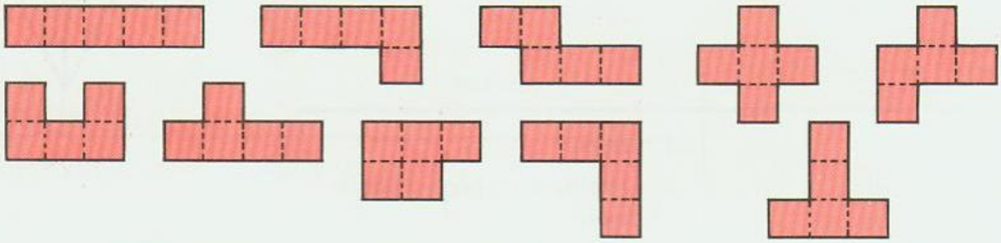


8. **Given:** $\overline{KM}$ is the perpendicular bisector of $\overline{JL}$.



Activity

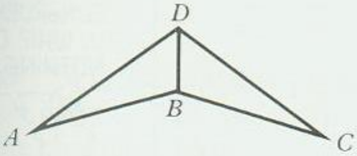
Make enlarged copies of these ten pentominoes and put them together to form a pair of squares identical in size and shape.



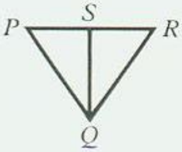
B.

Write a two-column proof for each of exercises 9–14.

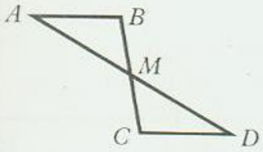
9. Given: BD bisects $\angle ADC$.
Prove: $\angle ADB \cong \angle BDC$



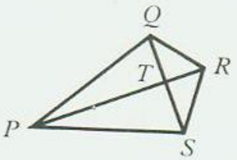
10. Given: $\overline{QS}$ is the perpendicular bisector of $\overline{PR}$.
Prove: S is the midpoint of $\overline{PR}$.



11. Given: M is the midpoint of $\overline{AD}$.
Prove: $\overline{AM} \cong \overline{MD}$

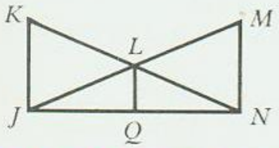


12. Given: $\overline{PR} \perp \overline{QS}$
Prove: $\angle PTQ \cong \angle PTS$

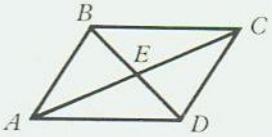


C.

13. Given: $\overline{LQ}$ is the perpendicular bisector of $\overline{JN}$.
Prove: $\overline{JQ} \cong \overline{QN}$



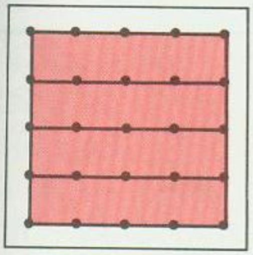
14. Given: $\overline{BD}$ bisects $\overline{AC}$.
Prove: $\overline{AE} \cong \overline{EC}$

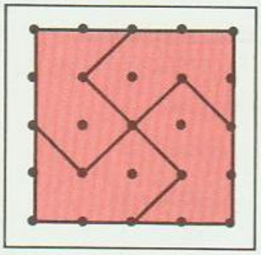


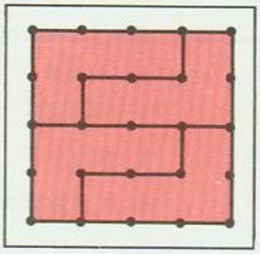
PROBLEM SOLVING

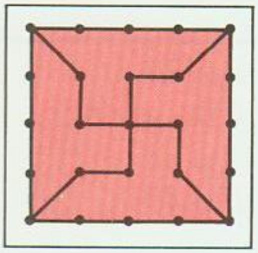
These pictures show four different ways to divide a square into four identical parts.

How many more different ways can you find? Show at least five of them on a geoboard or dot paper.

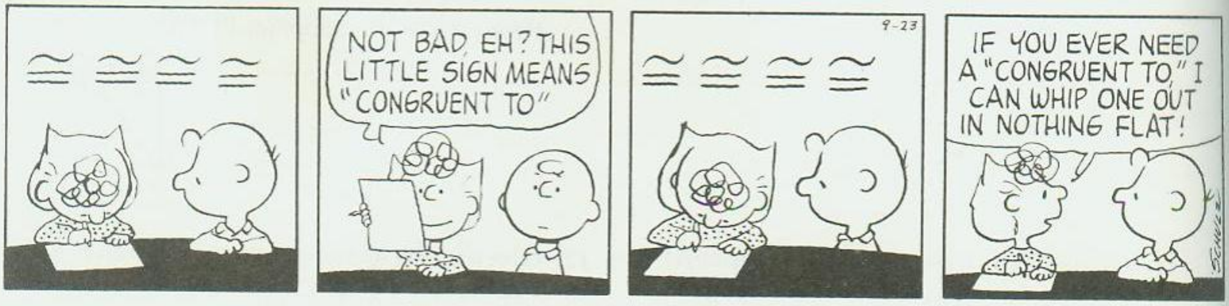








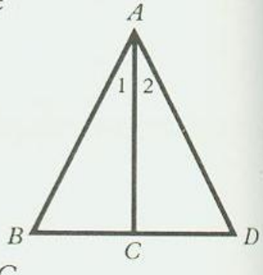
3–5 Proofs: Using Postulates and Definitions


≅ ≅ ≅ ≅
NOT BAD, EH? THIS LITTLE SIGN MEANS "CONGRUENT TO"
9-23
≅ ≅ ≅ ≅
IF YOU EVER NEED A "CONGRUENT TO," I CAN WHIP ONE OUT IN NOTHING FLAT!
SCHULZ

© 1980 United Feature Syndicate, Inc.

The definitions of angle bisector, segment bisector, perpendicular bisector, and midpoint can be used together with the congruence postulates to prove that two triangles are congruent.

To figure out how to prove that two triangles are congruent, it often helps to analyze the situation by starting with what is to be proved and working backwards. Study this example.



Problem: To prove that two triangles are congruent, as indicated.

Given: $\overline{AB} \cong \overline{AD}$
$\overline{AC}$ bisects $\angle BAD$.
Prove: $\triangle ABC \cong \triangle ADC$

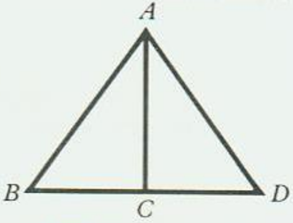
Analysis (how someone might figure out how to do it): "I want to prove that $\triangle ABC \cong \triangle ADC$. I could do this by using the SSS, SAS, or ASA Congruence Postulates. Which one? From the given information, I know that $\overline{AB} \cong \overline{AD}$. If I could show that $\angle 1 \cong \angle 2$ and $\overline{AC} \cong \overline{AC}$, I could use SAS. But $\overline{AC}$ bisects $\angle BAD$, so $\angle 1 \cong \angle 2$. Also, a segment is congruent to itself. I can do it!"

Proof

Statements	Reasons
1. $\overline{AB} \cong \overline{AD}$	1. Given
2. $\overline{AC}$ bisects $\angle BAD$.	2. Given
3. $\angle 1 \cong \angle 2$	3. Why?
4. $\overline{AC} \cong \overline{AC}$	4. A segment is congruent to itself.
5. $\triangle ABC \cong \triangle ADC$	5. SAS Congruence Postulate

Think through the *analysis* for each proof below. Then complete the proof.

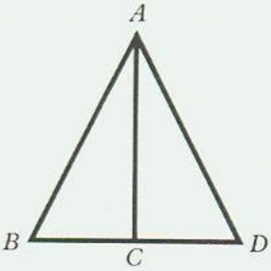
Example 1 **Given:** $\overline{AB} \cong \overline{AD}$
C is the midpoint of $\overline{BD}$.
Prove: $\triangle ABC \cong \triangle ADC$



Analysis: "Which congruence postulate should I aim for—ASA, SAS, or SSS? I can prove $\triangle ABC \cong \triangle ADC$ if I can show that three sides of $\triangle ABC$ are congruent respectively to three sides of $\triangle ADC$. I know from the 'given' that $\overline{AB} \cong \overline{AD}$. $\overline{AC} \cong \overline{AC}$, because a segment is congruent to itself. $\overline{BC} \cong \overline{CD}$ if C is a midpoint of $\overline{BD}$. It is! (from the 'given.') I can now write the proof."

Proof

Statements	Reasons
1. $\overline{AB} \cong \overline{AD}$	1. Given
2. C is the midpoint of $\overline{BD}$	2. Given
3. $\overline{BC} \cong \overline{CD}$	3. Why?
4. $\overline{AC} \cong \overline{AC}$	4. A segment is congruent to itself.
5. $\triangle ABC \cong \triangle ADC$	5. Why?



Example 2 **Given:** $\overline{AC}$ is the perpendicular bisector of $\overline{BD}$.
Prove: $\triangle ACB \cong \triangle ACD$

Analysis: "I can prove $\triangle ACB \cong \triangle ACD$ by using one of the congruence postulates. Let's try SAS. I know that $\overline{AC} \cong \overline{AC}$. Since $\overline{AC}$ is the perpendicular bisector of $\overline{BC}$, then $\angle ACB \cong \angle ACD$ and $\overline{BC} \cong \overline{CD}$. I can now write the proof!"

Proof

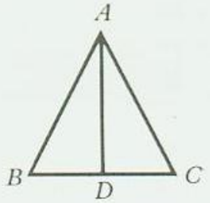
Statements	Reasons
1. $\overline{AC}$ bisects $\overline{BD}$.	1. Given
2. $\overline{BC} \cong \overline{CD}$	2. Definition of segment bisector
3. $\overline{AC} \perp \overline{BD}$	3. Given
4. $\angle ACB \cong \angle ACD$	4. Why?
5. $\overline{AC} \cong \overline{AC}$	5. Why?
6. $\triangle ACB \cong \triangle ACD$	6. SAS Postulate

EXERCISES

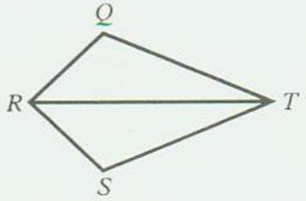
A.

In exercises 1–4 analyze the situation and indicate which of the three congruence postulates (SSS, SAS, or ASA) could be used to prove the triangles congruent.

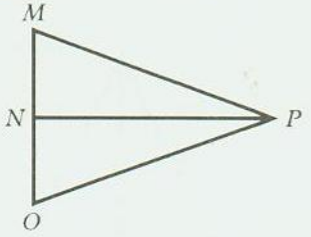
1. Given: $\overline{AD}$ bisects $\overline{BC}$.
$\overline{AB} \cong \overline{AC}$
Prove: $\triangle ABD \cong \triangle ACD$



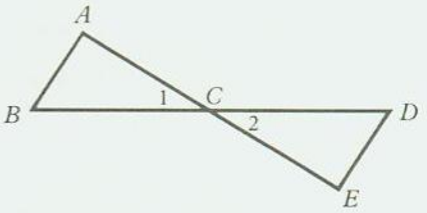
2. Given: $\overline{RT}$ bisects $\angle QRS$.
$\overline{RT}$ bisects $\angle QTS$.
Prove: $\triangle RTQ \cong \triangle RTS$



3. Given: $\overline{NP} \perp \overline{MO}$
$\overline{NP}$ bisects $\angle MPO$.
Prove: $\triangle MNP \cong \triangle ONP$

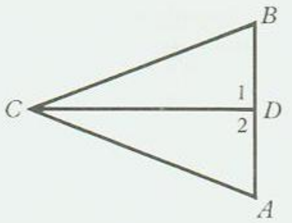


4. Given: $\overline{AE}$ and $\overline{BD}$ bisect each other.
$\angle 1 \cong \angle 2$
Prove: $\triangle ABC \cong \triangle EDC$



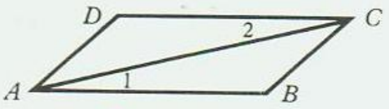
In exercises 5–7, supply the missing statements or reasons.

5. Given: $\overline{AD} \cong \overline{DB}$
$\overline{AB} \perp \overline{DC}$
Prove: $\triangle DAC \cong \triangle DBC$



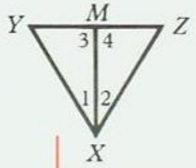
Statements	Reasons
1. $\overline{AD} \cong \overline{DB}$	1. Given
2. $\overline{AB} \perp \overline{DC}$	2. ?
3. $\angle 1 \cong \angle 2$	3. ?
4. ?	4. A segment is congruent to itself.
5. $\triangle DAC \cong \triangle DBC$	5. ?

6. Given: $\overline{AB} \cong \overline{CD}$
$\angle 1 \cong \angle 2$
Prove: $\triangle ABC \cong \triangle CDA$



Statements	Reasons
1. $\overline{AB} \cong \overline{CD}$	**1.** Given
2. $\angle 1 \cong \angle 2$	**2.** Given
3. ?	**3.** A segment is congruent to itself.
4. $\triangle ABC \cong \triangle CDA$	**4.** ?

7. Given: $\overline{XM} \perp \overline{YZ}$
$\overline{XM}$ bisects $\angle YXZ$.
Prove: $\triangle XYM \cong \triangle XZM$

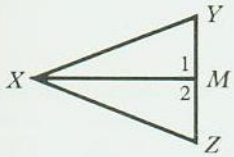


Statements	Reasons
1. $\overline{XM} \perp \overline{YZ}$	**1.** Given
2. ?	**2.** Definition of perpendicular lines
3. $\overline{XM}$ bisects $\angle YXZ$	**3.** ?
4. $\angle 1 \cong \angle 2$	**4.** ?
5. $\overline{XM} \cong \overline{XM}$	**5.** ?
6. $\triangle XYM \cong \triangle XZM$	**6.** ?

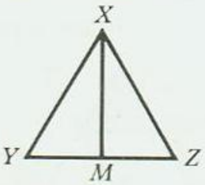
B.

In exercises 8–17 prove that the triangles are congruent.

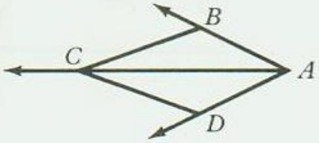
8. Given: $\overline{XM}$ bisects $\angle YXZ$.
$\angle 1 \cong \angle 2$
Prove: $\triangle XMY \cong \triangle XMZ$



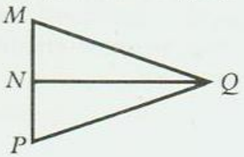
9. Given: $\overline{XM} \perp \overline{YZ}$
$\overline{YM} \cong \overline{MZ}$
Prove: $\triangle XYM \cong \triangle XZM$



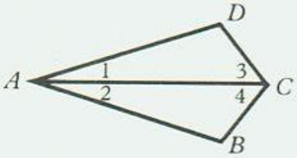
10. Given: $\overline{AC}$ is the bisector of $\angle BAD$.
$\overline{AB} \cong \overline{AD}$
Prove: $\triangle BAC \cong \triangle DAC$



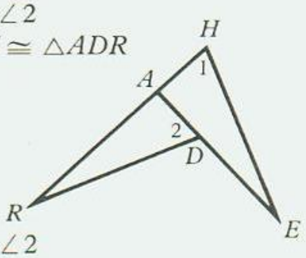
11. Given: $\overline{QM} \cong \overline{QP}$
$\overline{QN}$ bisects $\overline{MP}$.
Prove: $\triangle QNM \cong \triangle QNP$



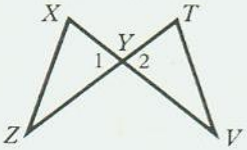
12. Given: $\overline{AC}$ bisects $\angle DAB$.
$\overline{AC}$ bisects $\angle DCB$.
Prove: $\triangle ACD \cong \triangle ACB$



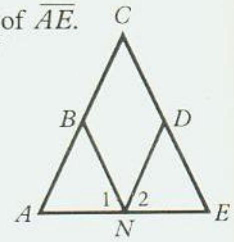
13. Given: $\overline{HR} \perp \overline{AE}$
$\overline{AH} \cong \overline{AD}$
$\angle 1 \cong \angle 2$
Prove: $\triangle AHE \cong \triangle ADR$



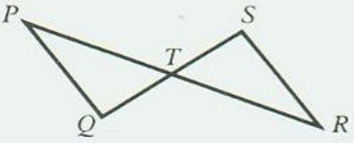
14. Given: $\angle 1 \cong \angle 2$
$\overline{XY} \cong \overline{TY}$
$\overline{ZY} \cong \overline{VY}$
Prove: $\triangle XYZ \cong \triangle TYV$



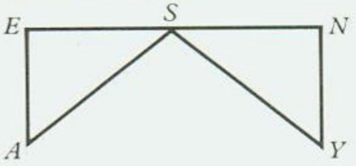
15. Given: $\angle 1 \cong \angle 2$
$\angle A \cong \angle E$
N is the midpoint of $\overline{AE}$.
Prove: $\triangle ABN \cong \triangle EDN$



16. Given: $\overline{PQ} \cong \overline{SR}$
$\overline{PR}$ bisects $\overline{SQ}$.
$\overline{SQ}$ bisects $\overline{PR}$.
Prove: $\triangle PQT \cong \triangle RST$

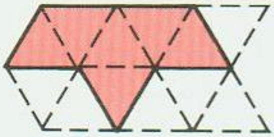


17. Given: $\angle E \cong \angle N$
S bisects $\overline{EN}$.
$\overline{EA} \cong \overline{NY}$
Prove: $\triangle EAS \cong \triangle NYS$

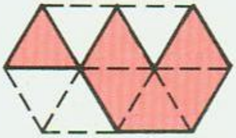


Activity

A *hexiamond* is a polygon which bounds six equilateral triangles.

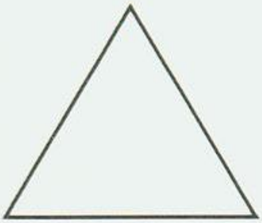


This is a hexiamond.



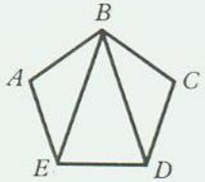
This is not a hexiamond.

Trace and cut six copies of this equilateral triangle from heavy paper or cardboard. Experiment with your six equilateral triangles and discover twelve hexiamonds different in shape. Draw each of these twelve polygons.

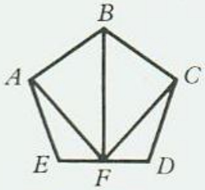


C.

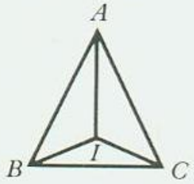
18. **Given:** $ABCDE$ is a regular pentagon.
Prove: $\triangle AEB \cong \triangle CDB$



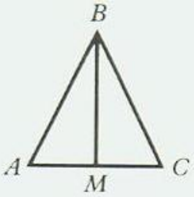
19. **Given:** $\overline{BF}$ bisects $\angle ABC$.
$ABCDE$ is a regular pentagon.
Prove: $\triangle ABF \cong \triangle CBF$



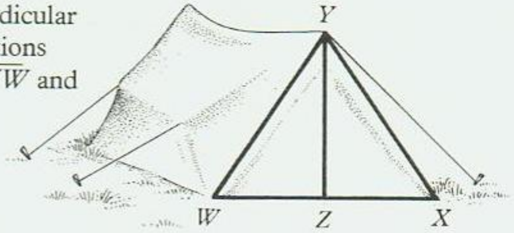
20. **Given:** $\overline{AB} \cong \overline{AC}$
$\overline{AI}$ bisects $\angle BAC$
Prove: $\triangle AIB \cong \triangle AIC$



21. **Given:** $\overline{AB} \cong \overline{BC}$
M is midpoint of $\overline{AC}$
Prove: $\angle A \cong \angle C$



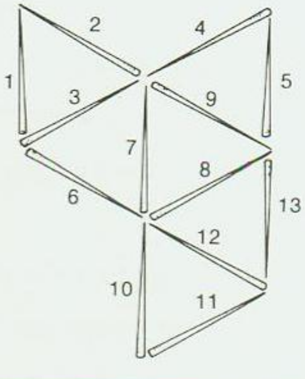
22. Pole $\overline{YZ}$ of a tent is placed perpendicular to level ground. What other conditions must be met to insure that sides $\overline{YW}$ and $\overline{YX}$ are equal in length?



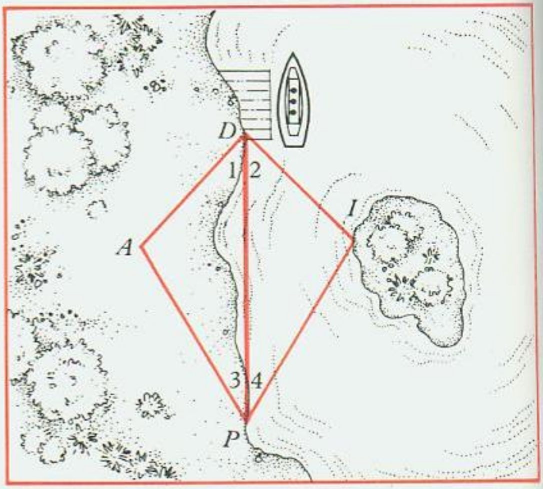
PROBLEM SOLVING

The picture shows 13 toothpicks arranged to show six triangles. Which three toothpicks can be removed so that the remaining toothpicks show three triangles?

Can you make a toothpick problem similar to this?



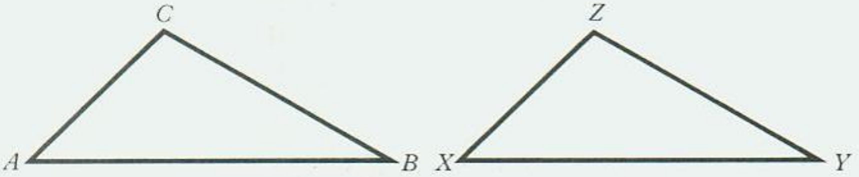
3–6 Proving Segments and Angles Congruent



An ingenious geometry student (who was a good swimmer, but not *too* good) used this method to find the distance from the dock to the island:

Pick a second point, P, on the shore. Make $\angle 1 \cong \angle 2$. Make $\angle 3 \cong \angle 4$. Locate, by sight, the intersection point A of the sides of angles 1 and 3. Why is the distance from the dock to the island (DI) the same as DA?

To answer this question, recall the definition of congruent triangles (page 85).



$\triangle ABC \cong \triangle XYZ$ means these six things are true.

$\overline{AB} \cong \overline{XY} \quad \overline{AC} \cong \overline{XZ} \quad \overline{BC} \cong \overline{YZ} \quad \angle A \cong \angle X \quad \angle B \cong \angle Y \quad \angle C \cong \angle Z$

We often prove that a pair of segments or angles are congruent by first proving that a pair of triangles are congruent. We can then use the definition of congruent triangles to conclude that the parts of the triangles that correspond are congruent.

In the dock-island situation above $\triangle DAP \cong \triangle DIP$ by the ASA Postulate. Since $\overline{DA}$ and $\overline{DI}$ are corresponding parts of these triangles, $\overline{DA} \cong \overline{DI}$.

To prove segments or angles congruent, you often can

1. select triangles that contain these segments (or angles);
2. prove the triangles congruent;
3. conclude that the desired corresponding parts of these triangles are congruent.

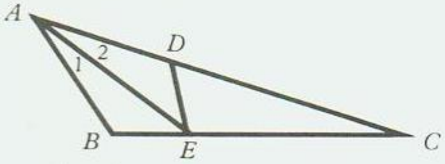
When writing proofs, we give the following reason for Step 3.

Corresponding Parts of Congruent Triangles are Congruent

Abbreviation: Corresp. parts of $\cong$ $\triangle$s are $\cong$ or **CPCTC**

This procedure is illustrated in the following two examples.

Example 1 Given: $\overline{AB} \cong \overline{AD}$, $\angle 1 \cong \angle 2$
Prove: $\overline{BE} \cong \overline{DE}$

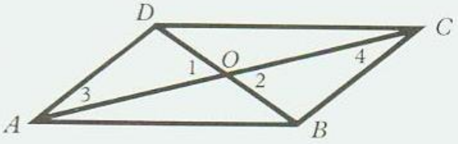


Analysis: "I can prove $\overline{BE} \cong \overline{DE}$ if I can find a pair of congruent triangles that contain these segments. $\triangle ABE$ and $\triangle ADE$ look promising. Which congruence postulate can I use to prove them congruent? Try SAS. Since I know that $\angle 1 \cong \angle 2$, $\overline{AB} \cong \overline{AD}$, and $\overline{AE} \cong \overline{AE}$, I can prove the triangles congruent and conclude that $\overline{BE} \cong \overline{DE}$.

Proof

Statements	Reasons
1. $\overline{AB} \cong \overline{AD}$	**1.** Given
2. $\angle 1 \cong \angle 2$	**2.** Given
3. $\overline{AE} \cong \overline{AE}$	**3.** Why?
4. $\triangle ABE \cong \triangle ADE$	**4.** SAS Postulate
5. $\overline{BE} \cong \overline{DE}$	**5.** Corresp. parts of $\cong$ $\triangle$s are $\cong$.

Example 2 Given: $\overline{AC}$ and $\overline{BD}$ bisect each other.
$\angle 1 \cong \angle 2$
Prove: $\angle 3 \cong \angle 4$



Analysis: "$\angle 3$ and $\angle 4$ are in $\triangle AOD$ and $\triangle BOC$ respectively, and in $\triangle ADC$ and $\triangle ABC$ respectively. From the given information it appears I should prove $\triangle AOD$ congruent to $\triangle COB$. (Why?) Then I can conclude that $\angle 3$ is congruent to $\angle 4$."

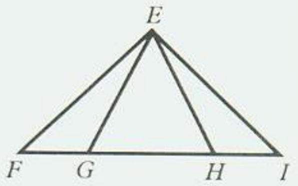
Proof

Statements	Reasons
1. $\overline{AC}$ and $\overline{BD}$ bisect each other.	**1.** Given
2. $\overline{AO} \cong \overline{CO}$; $\overline{OB} \cong \overline{OD}$	**2.** Definition of segment bisector
3. $\angle 1 \cong \angle 2$	**3.** Given
4. $\triangle AOD \cong \triangle COB$	**4.** SAS Postulate
5. $\angle 3 \cong \angle 4$	**5.** CPCTC

EXERCISES

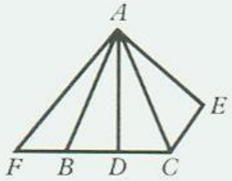
A.

In exercises 1–5, indicate which triangles could be proven congruent to establish the indicated fact. (In some cases there may be more than one pair of triangles that are correct.)



(Exs. 2, 3)

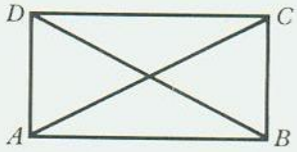
1. Prove: $\overline{AB} \cong \overline{AC}$



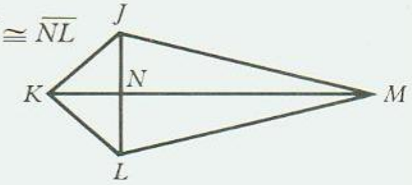
2. Prove: $\angle F \cong \angle I$

3. Prove: $\overline{EG} \cong \overline{EH}$

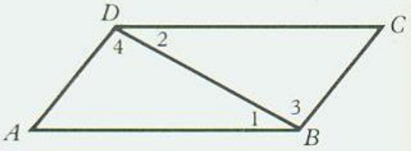
4. Prove: $\overline{AD} \cong \overline{BC}$



5. Prove: $\overline{JN} \cong \overline{NL}$



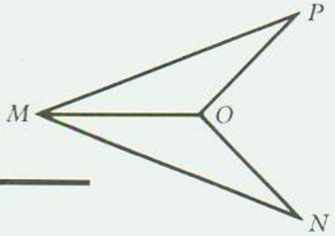
In exercises 6–8 provide the missing statements and reasons.



6. Given: $\angle 1 \cong \angle 2$
$\angle 3 \cong \angle 4$
Prove: $\angle A \cong \angle C$

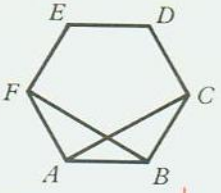
Statements	Reasons
1. $\angle 1 \cong \angle 2$	1. ?
2. ?	2. Given
3. $\overline{DB} \cong \overline{DB}$	3. A segment is congruent to itself.
4. $\triangle ADB \cong \triangle CBD$	4. ?
5. $\angle A \cong \angle C$	5. ?

7. Given: $\overline{MN} \cong \overline{MP}$
$\overline{NO} \cong \overline{OP}$
Prove: $\overline{MO}$ bisects $\angle NMP$.



Statements	Reasons
1. $\overline{MN} \cong \overline{MP}$; $\overline{NO} \cong \overline{OP}$	1. ?
2. ?	2. ?
3. $\triangle MNO \cong \triangle MPO$	3. ?
4. ?	4. CPCTC
5. $\overline{MO}$ bisects $\angle NMP$.	5. ?

8. Given: *ABCDEF* is a regular hexagon.
Prove: $\overline{AC} \cong \overline{BF}$

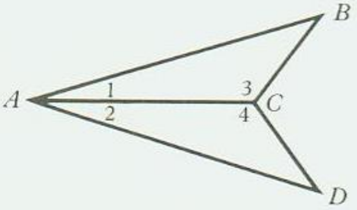


Statements	Reasons
1. *ABCDEF* is a regular hexagon.	1. Given
2. $\overline{AF} \cong \overline{BC}$	2. ?
3. ?	3. A segment is congruent to itself.
4. $\angle FAB \cong \angle ABC$	4. ?
5. $\triangle FAB \cong \triangle CBA$	5. ?
6. ?	6. CPCTC

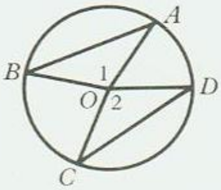
B.

For the remaining exercises, write complete two-column proofs.

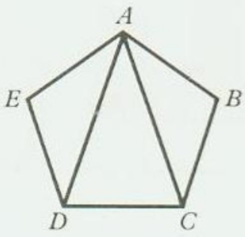
9. Given: $\angle 1 \cong \angle 2$
$\angle 3 \cong \angle 4$
Prove: $\overline{BC} \cong \overline{CD}$



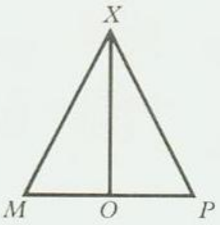
10. Given: $\overline{OA}$, $\overline{OB}$, $\overline{OC}$, $\overline{OD}$ are all congruent.
$\angle 1 \cong \angle 2$
Prove: $\overline{AB} \cong \overline{CD}$



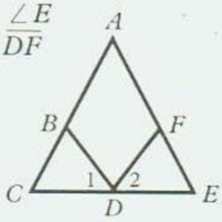
11. Given: *ABCDE* is a regular pentagon.
Prove: $\triangle ADC$ is isosceles.



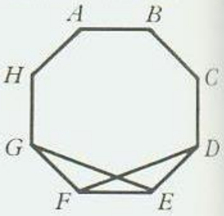
12. Given: $\overline{XO}$ is the perpendicular bisector of $\overline{MP}$.
Prove: $\triangle XMP$ is isosceles.



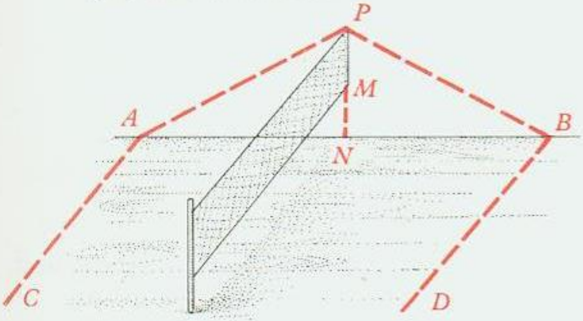
13. **Given:** $\angle 1 \cong \angle 2$
D bisects $\overline{CE}$.
$\angle C \cong \angle E$
Prove: $\overline{BD} \cong \overline{DF}$



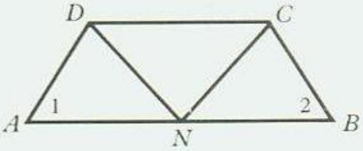
14. **Given:** $ABCDEFGH$ is a regular octagon.
Prove: $\overline{DF} \cong \overline{GE}$



15. In a gym one end of a volleyball net is hung from bolts secured to a wall at P and M. Each point in the plane of the net is an equal distance from the two base lines $\overline{AC}$ and $\overline{BD}$. Why is $\overleftrightarrow{PM}$ perpendicular to $\overline{AB}$?



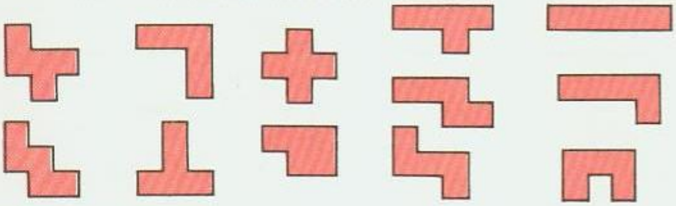
16. **Given:** $\overline{AD} \cong \overline{BC}$
$\angle 1 \cong \angle 2$
N is the midpoint of $\overline{AB}$.
Prove: $\triangle CND$ is isosceles.

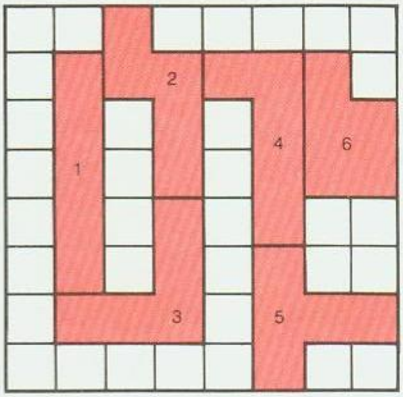


Activity

Make these 12 pentominoe shapes from squares the size of those on a checker board.

Six pentominoe pieces have been arranged so that none of the remaining six pieces will fit on the checker board without overlapping. Do this with a different collection of six pieces.

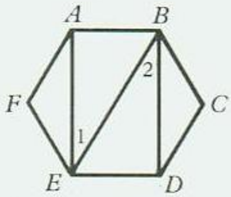




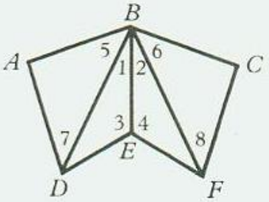
C.

In exercises 17-20 you may also need to prove more than one pair of triangles congruent.

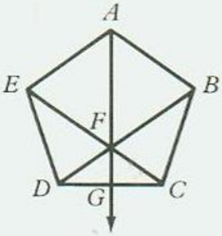
17. Given: $ABCDEF$ is a regular hexagon.
Prove: $\angle 1 \cong \angle 2$
(*Hint:* First prove that $\overline{AE} \cong \overline{BD}$.)



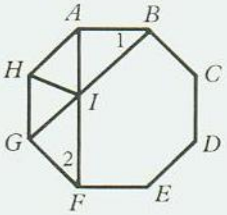
18. Given: $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$
$\angle 5 \cong \angle 6$, $\angle 7 \cong \angle 8$
Prove: $\angle A \cong \angle C$



19. Given: $ABCDE$ is a regular pentagon.
$\overrightarrow{AG}$ bisects $\angle EAB$ and is the perpendicular bisector of $\overline{DC}$.
Prove: $\triangle BCF \cong \triangle EDF$

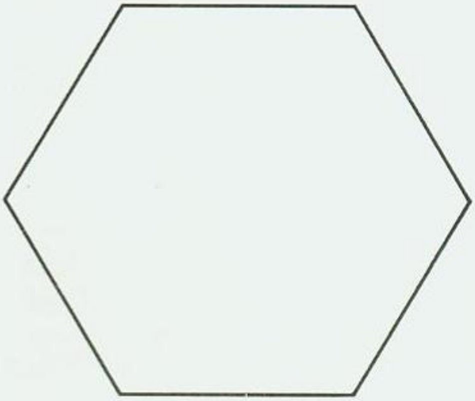


20. Given: $ABCDEFGH$ is a regular octagon.
$\overline{AF} \perp \overline{AB}$; $\overline{BG} \perp \overline{GF}$
$\angle 1 \cong \angle 2$
Prove: $\triangle AHI \cong \triangle GHI$

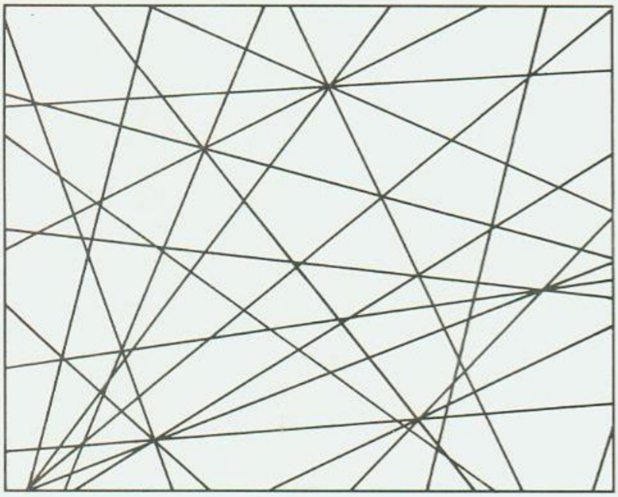


PROBLEM SOLVING

Show how to cut a regular hexagon into 18 congruent kites. A kite is a quadrilateral with exactly two pairs of congruent adjacent sides. (*Hint:* Draw all the diagonals of the hexagon. Then draw six more short segments, strategically located.)



3–7 Proofs: Overlapping Triangles

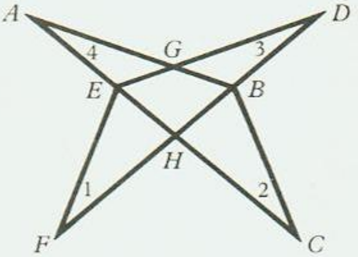


A popular "puzzle problem" requests that you find a pair of congruent equilateral triangles in this figure and trace them on your paper. Can you solve it?

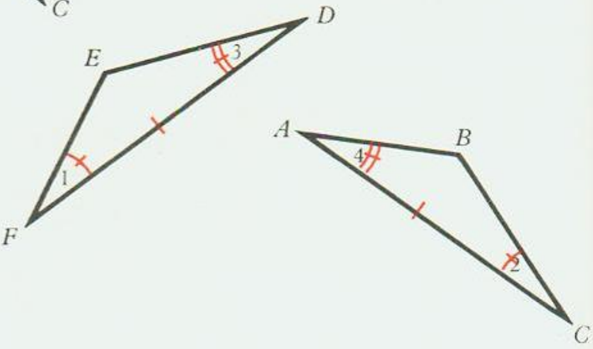
In proofs several triangles in a figure often overlap each other, making it difficult to visualize the triangles that would be the most useful to prove congruent. Sometimes it is helpful to pictorially or mentally separate the triangles to aid in analyzing the proof, as indicated in the example below.

Example 1

Given: $\angle 1 \cong \angle 2$
$\overline{AC} \cong \overline{DF}$
$\angle 3 \cong \angle 4$
Prove: $\overline{EF} \cong \overline{BC}$



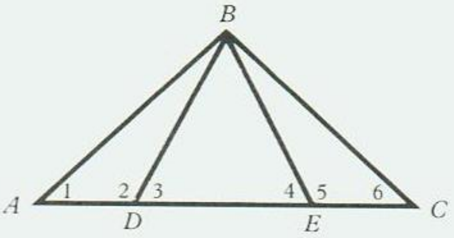
Analysis: "I need to choose a pair of triangles that contain $\overline{EF}$ and $\overline{BC}$. How about $\triangle EFH$ and $\triangle BCH$? I can't prove them congruent. Let's try $\triangle EFD$ and $\triangle BCA$! They are congruent because of the ASA postulate and I can conclude that $\overline{EF} \cong \overline{BC}$."



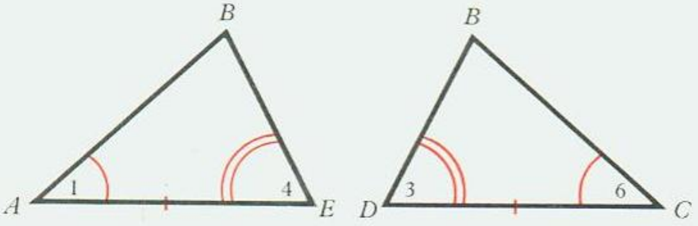
You will be asked to complete this proof as an exercise.

Example 2

Given: $\angle 1 \cong \angle 6$
$\angle 3 \cong \angle 4$
$\overline{AE} \cong \overline{CD}$
Prove: $\angle ABE \cong \angle CBD$



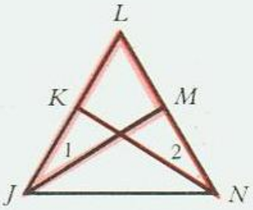
Analysis: "Overlapping triangles $\triangle ABE$ and $\triangle CBD$ contain two pairs of congruent angles. The included sides are also congruent. I can use ASA to prove it."



Sometimes it is useful to use colored pencils to outline the overlapping triangles. Study the following example.

Example 3

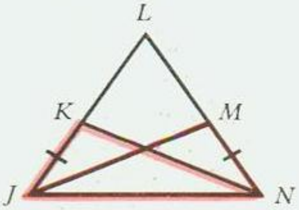
Given: $\triangle LJN$ is isosceles with $\overline{JL} \cong \overline{LN}$.
$\angle 1 \cong \angle 2$
Prove: $\overline{LK} \cong \overline{LM}$



Analysis: "I can prove $\overline{LK} \cong \overline{LM}$ if these segments are corresponding parts of congruent triangles. Let's try $\triangle LKN$ and $\triangle LMJ$. I know that $\angle 1 \cong \angle 2$ and that $\angle JLN$ is congruent to itself. Aha! I know that $\triangle JLN$ is isosceles. So the triangles are congruent by ASA. I can prove it!"

Example 4

Given: $\overline{JK} \cong \overline{NM}$, $\angle KJN \cong \angle MNJ$
Prove: $\overline{KN} \cong \overline{MJ}$



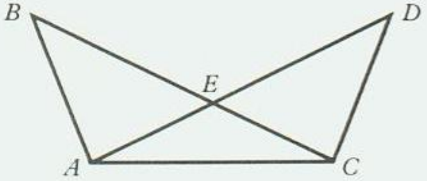
Analysis: "$\overline{KN}$ and $\overline{MJ}$ are corresponding parts in $\triangle LKN$ and $\triangle LMJ$ just like Example 3. But the given information is part of $\triangle KJN$ and $\triangle MNJ$. Since $\overline{JN}$ is congruent to itself, I can prove $\triangle KJN \cong \triangle MNJ$ by SAS."

EXERCISES

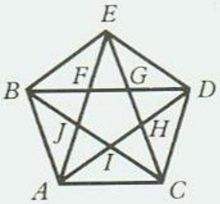
A.

In exercises 1 and 2 name all pairs of triangles (nonoverlapping and overlapping) that appear congruent. Use vertices A and B in one triangle and vertices C and D in the other triangle.

1.



2.



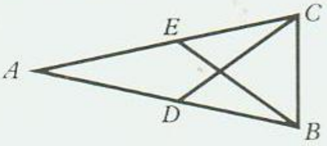
B.

3. Write a complete two-column proof for Example 1.

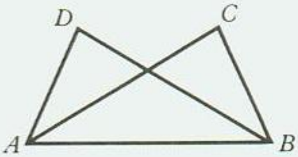
4. Write a complete two-column proof for Example 3.

For exercises 5–10 write a complete two-column proof.

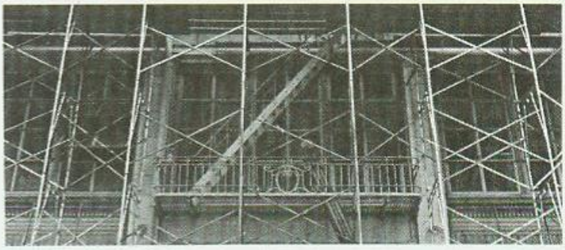
5. Given: $\overline{BD} \cong \overline{CE}$
$\angle ABC \cong \angle ACB$
Prove: $\overline{BE} \cong \overline{CD}$



6. Given: $\angle DAB \cong \angle CBA$
$\angle DBA \cong \angle CAB$
Prove: $\overline{AD} \cong \overline{BC}$

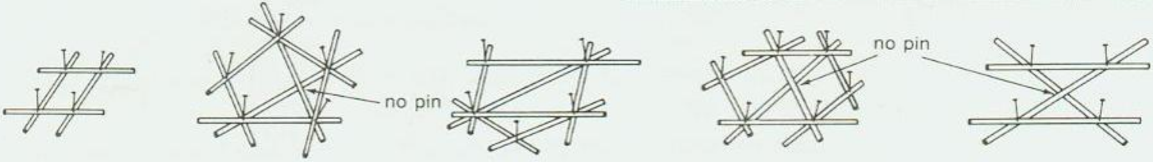


Activity

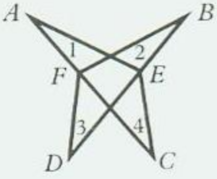


Triangles are used in scaffolding because they are rigid figures. Three straws pinned together will hold its shape even if picked up and moved around.

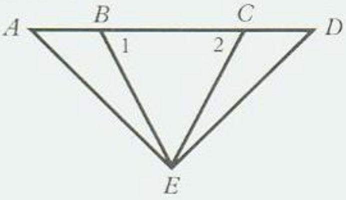
Which of the frameworks do you think are rigid? Build models to check your answers.



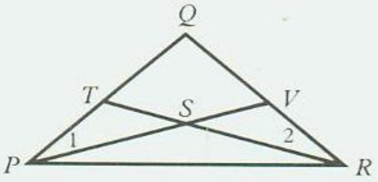
7. Given: $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$
$\overline{AC} \cong \overline{BD}$
Prove: $\overline{AE} \cong \overline{BF}$



8. Given: $\angle 1 \cong \angle 2$, $\overline{BE} \cong \overline{EC}$
$\angle AEC \cong \angle BED$
Prove: $\overline{AE} \cong \overline{DE}$

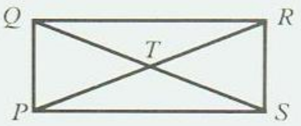


9. Given: $\angle 1 \cong \angle 2$, $\overline{PQ} \cong \overline{RQ}$
$\overline{PV} \cong \overline{TR}$
Prove: $\overline{QT} \cong \overline{QV}$



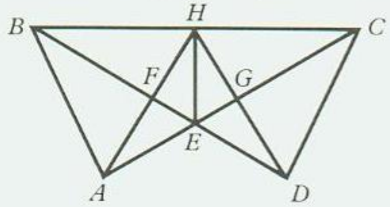
10. Given: Figure $PQRS$ with $\overline{PQ} \cong \overline{RS}$, $\angle RQP$ and $\angle QRS$ are right angles.
Prove: $\overline{QS} \cong \overline{RP}$

(i.e., the diagonals are congruent.)

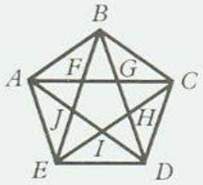


C.

11. Given: $\overline{HF} \perp \overline{BD}$, $\overline{HG} \perp \overline{AC}$
$\overline{HF} \cong \overline{HG}$
Prove: $\overline{AG} \cong \overline{DF}$

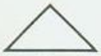


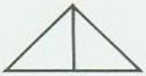
12. Given: Regular pentagon $ABCDE$
Prove: $\overline{AD} \cong \overline{EB}$

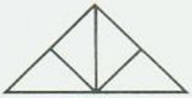


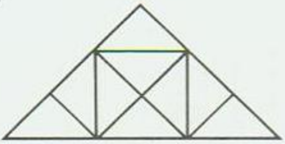
PROBLEM SOLVING

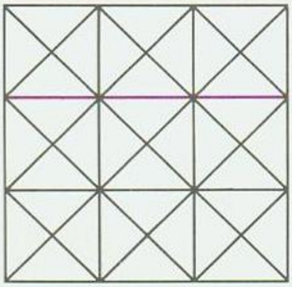
How many different triangles are there in this figure that are congruent to the lettered triangles below?


A


B


C


D



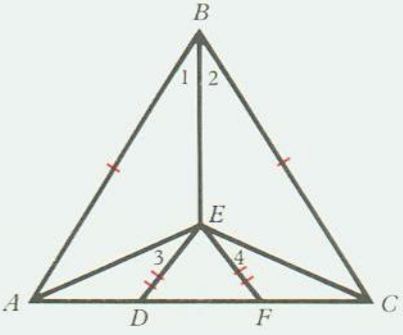
Do you see other types of triangles in the figure? If so, draw them.

3–8 Proofs: Chains of Congruences

To write certain proofs, one pair of triangles must be proven congruent in order to provide the information needed to prove a second pair of triangles congruent.

Study the example below and supply the missing steps.

Example **Given:** $\overline{AB} \cong \overline{CB}$
$\overline{ED} \cong \overline{EF}$
$\angle 1 \cong \angle 2$
$\angle 3 \cong \angle 4$
Prove: $\overline{AD} \cong \overline{CF}$



Analysis: "I could prove $\overline{AD} \cong \overline{CF}$ if I could prove $\triangle AED \cong \triangle CEF$. But not enough information is given to prove this. If I knew that $\overline{AE} \cong \overline{CE}$, I could prove these triangles congruent by SAS. But $\overline{AE}$ and $\overline{CE}$ are corresponding parts of $\triangle ABE$ and $\triangle CBE$, and I can prove these triangles congruent. I can do it!"

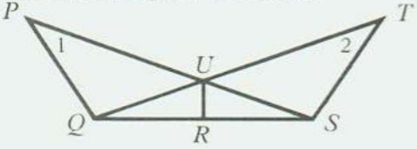
Proof

Statements	Reasons
1. $\overline{AB} \cong \overline{CB}$	**1.** Given
2. $\angle 1 \cong \angle 2$	**2.** Why?
3. $\overline{BE} \cong \overline{BE}$	**3.** Why?
4. $\triangle ABE \cong \triangle CBE$	**4.** SAS postulate
5. $\overline{AE} \cong \overline{CE}$	**5.** Why?
6. $\angle 3 \cong \angle 4$	**6.** Given
7. $\overline{DE} \cong \overline{FE}$	**7.** Given
8. $\triangle AED \cong \triangle CEF$	**8.** Why?
9. $\overline{AD} \cong \overline{CF}$	**9.** CPCTC

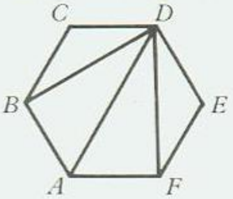
Note that Steps 1–4 prove a pair of triangles congruent. Steps 5–8 use this first congruence to prove a second pair of triangles congruent.

EXERCISES

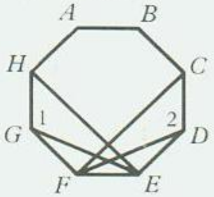
1. **Given:** $\angle 1 \cong \angle 2$, $\overline{UR}$ bisects $\overline{QS}$,
$\overline{PU} \cong \overline{TU}$, $\angle PUQ \cong \angle TUS$
Prove: $\triangle QRU \cong \triangle SRU$



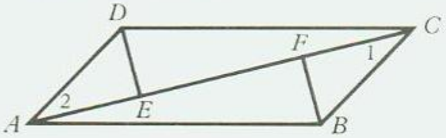
2. **Given:** $ABCDEF$ is a regular hexagon.
Prove: $\triangle ABD \cong \triangle AFD$



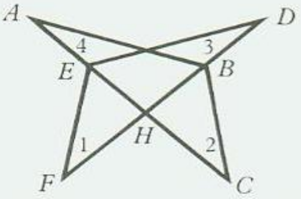
3. **Given:** $ABCDEFGH$ is a regular octagon.
$\angle 1 \cong \angle 2$
Prove: $\overline{CF} \cong \overline{HE}$



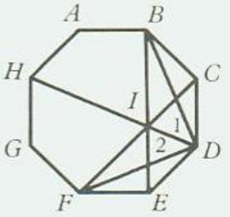
4. **Given:** $\angle 1 \cong \angle 2$
$\angle AED \cong \angle CFB$
$\overline{AE} \cong \overline{CF}$
Prove: $\overline{AB} \cong \overline{CD}$



5. **Given:** $\overline{EH} \cong \overline{BH}$
$\overline{AH} \cong \overline{DH}$
$\overline{AC} \cong \overline{DF}$
$\angle 1 \cong \angle 2$
Prove: $\overline{EF} \cong \overline{BC}$



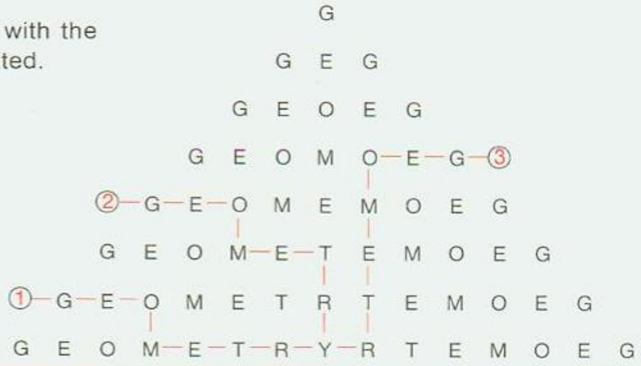
6. **Given:** $ABCDEFGH$ is a regular octagon.
$\overline{DH}$ bisects $\angle CDE$.
$\angle 1 \cong \angle 2$
Prove: $\triangle BCI \cong \triangle FEI$



PROBLEM SOLVING

How many ways can you spell "Geometry" with the arrangement shown? Three ways are indicated.

(*Hint:* How many ways can you spell a two-letter word with a similar arrangement? A three-letter word? A four-letter word? Find a pattern.)



Important Ideas—Chapter 3

Terms

Congruent triangles (p. 85)

Postulates

SAS Congruence Postulate. If two sides and the included angle of one triangle are congruent respectively to two sides and the included angle of another triangle, then the two triangles are congruent.

ASA Congruence Postulate. If two angles and the included side of one triangle are congruent respectively to two angles and the included side of another triangle, then the two triangles are congruent.

SSS Congruence Postulate. If all three sides of one triangle are congruent respectively to all three sides of another triangle, then the two triangles are congruent.

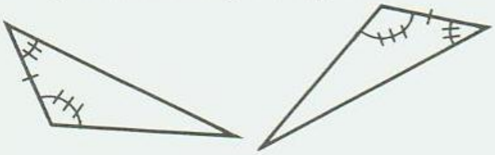
Statements

Corresponding parts of congruent triangles are congruent.

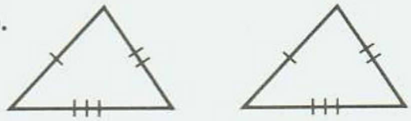
Chapter 3—Review

1. Given that $\triangle BAC \cong \triangle QRP$, identify the corresponding sides and angles.
2. In the following cases, state which congruence postulate could be used to prove the triangles congruent.

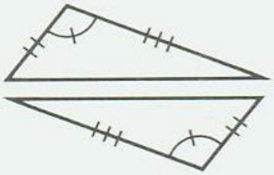
a.



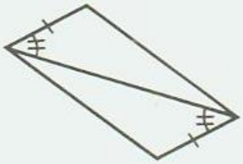
b.



c.

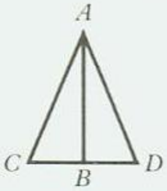


d.

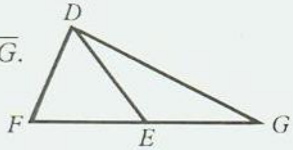


3. In each of the following cases, draw a conclusion based on the given information.

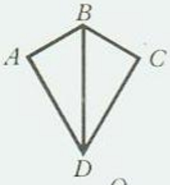
a. Given: $\overline{AB}$ is perpendicular to $\overline{CD}$.



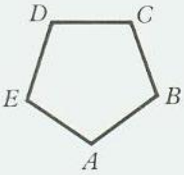
b. Given: E is the midpoint of $\overline{FG}$.



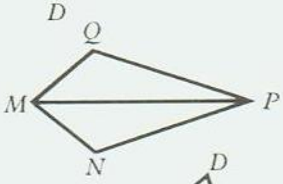
c. Given: $\overline{BD}$ bisects both $\angle B$ and $\angle D$.



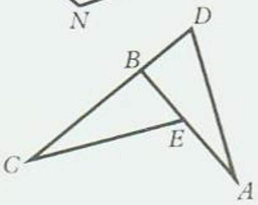
d. Given: $ABCDE$ is a regular pentagon.



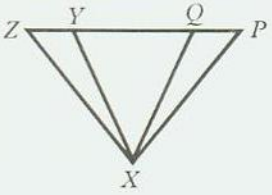
4. **Given:** $\overline{MP}$ bisects $\angle QMN$.
$\overline{MP}$ bisects $\angle QPN$.
Prove: a. $\triangle MQP \cong \triangle MNP$
b. $\angle Q \cong \angle N$



5. **Given:** $\overline{AB} \perp \overline{CD}$
$\overline{BE} \cong \overline{BD}$
$\overline{BC} \cong \overline{BA}$
Prove: $\overline{CE} \cong \overline{AD}$



6. **Given:** $\overline{ZQ} \cong \overline{PY}$
$\angle Z \cong \angle P$
$\overline{ZX} \cong \overline{PX}$
Prove: $\overline{XY} \cong \overline{XQ}$



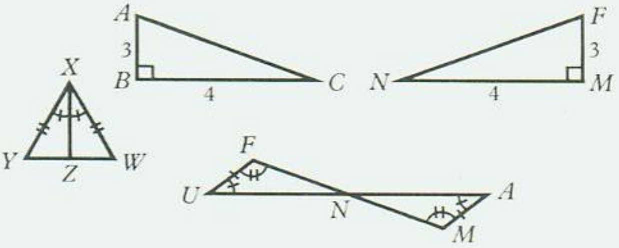
Chapter 3—Test

1. Use the figures to complete the statements.

   a. $\triangle ABC \cong \underline{\ ?\ }$

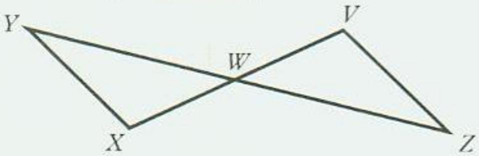
   b. $\triangle XYZ \cong \underline{\ ?\ }$

   c. $\triangle FUN \cong \underline{\ ?\ }$

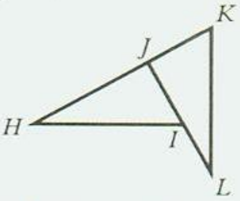


2. In the following cases, draw a conclusion based on the given information.

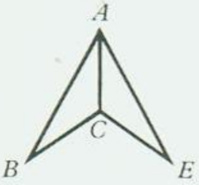
   a. **Given:** W is the midpoint of $\overline{YZ}$ and $\overline{XV}$.



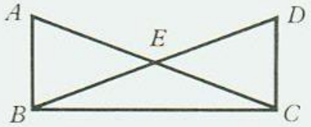
   b. **Given:** $\overline{JI} \perp \overline{HK}$



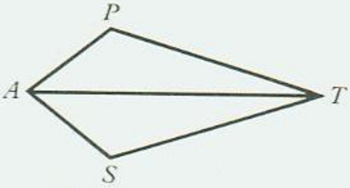
   c. **Given:** $\overline{AC}$ bisects $\angle BAE$



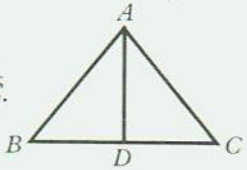
   d. **Given:** $\overline{AC}$ bisects $\overline{BD}$
   $\overline{BD}$ bisects $\overline{AC}$



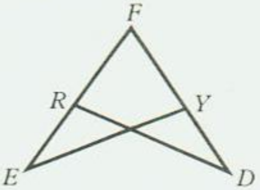
3. **Given:** $\overline{PA} \cong \overline{SA}$
   $\overline{PT} \cong \overline{TS}$
   Prove: a. $\triangle PAT \cong \triangle SAT$
   b. $\angle P \cong \angle S$



4. **Given:** $\overline{AD} \perp \overline{BC}$
   D is the midpoint of $\overline{BC}$.
   Prove: $\overline{AB} \cong \overline{AC}$



5. **Given:** $\overline{FE} \cong \overline{FD}$
   $\angle E \cong \angle D$
   Prove: $\overline{EY} \cong \overline{DR}$



Cumulative Review (Chapters 1–3)

1. Mark the following as true or false.
   - **a.** Postulates are statements to be proved.
   - **b.** A ray can contain a line.
   - **c.** Theorems are proved using deductive reasoning.
2. Draw a triangle ABC so that $\angle B$ is obtuse. Construct a line through B perpendicular to $\overline{AC}$.

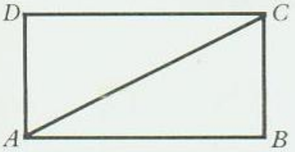
In exercises 3 and 4 classify the reasoning as inductive or deductive reasoning.

3. $\angle 1$ and $\angle 2$ are called right angles.
   Right angles are congruent.
   $\angle 1$ and $\angle 2$ are congruent.
4. $ABCD$ is a square and has perpendicular diagonals.
   $EFGH$ is a square and has perpendicular diagonals.
   All squares have perpendicular diagonals.

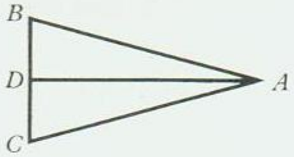
In exercises 5 and 6 state the correct conclusion or write "no conclusion possible."

5. If $\overline{AB} \perp \overline{CD}$ then $\overline{AB}$ and $\overline{CD}$ determine a plane.
   $\overline{AB}$ and $\overline{CD}$ determine a plane.
   Therefore, _?_.
6. If $ABCD$ is a square, then $\overline{AB} \parallel \overline{CD}$.
   $ABCD$ is a square.
   Therefore, _?_.
7. Consider the statement,
   If two lines are parallel, then they are not perpendicular.
   - **a.** Write the converse statement.
   - **b.** Write the inverse statement.
   - **c.** Write the contrapositive statement.
   - **d.** Give counterexamples for the statements which are not true.

8. **Given:** $\overline{AD} \cong \overline{BC}$, $\overline{AB} \cong \overline{DC}$
   Prove: $\triangle ABC \cong \triangle CDA$



9. **Given:** $\overline{AB} \cong \overline{AC}$, $\overline{AD}$ bisects $\angle BAC$.
   Prove: $\overline{BD} \cong \overline{CD}$.

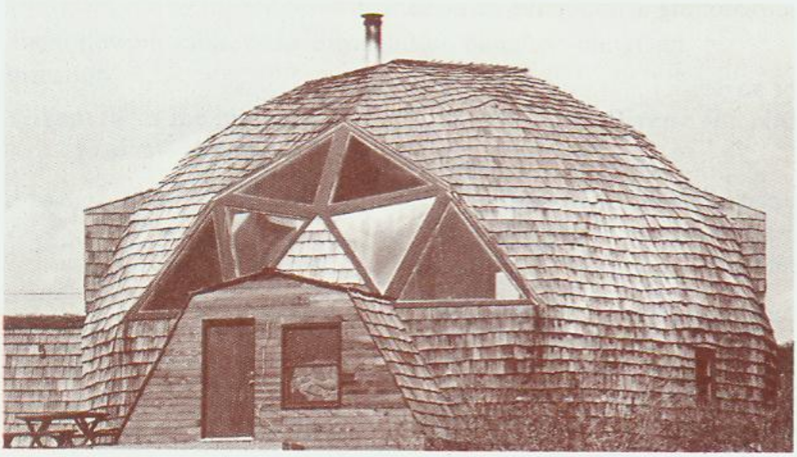


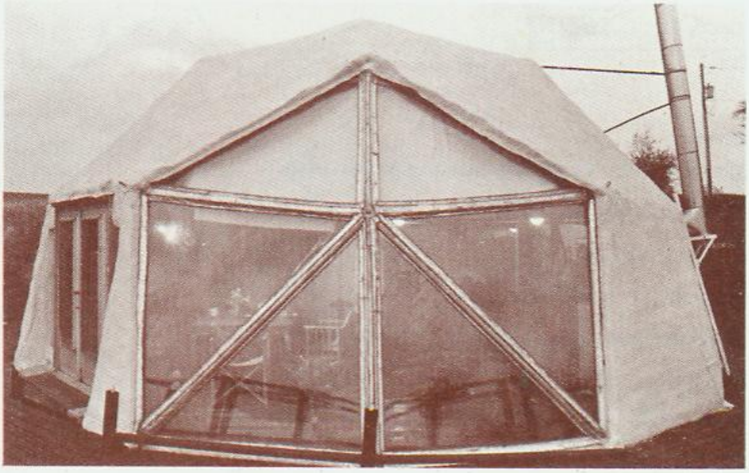
Geometry In Our World

Architecture: Geodesic Domes

The geodesic dome was introduced by R. Buckminster Fuller. His plans for one kind of dome, called a *sun dome,* can be found in the May 1966 issue of *Popular Science.*

Domes of different sizes and shapes have been constructed from a variety of materials. Domes have been used as greenhouses, pool covers, and even houses. Shown below are a dome house in Colorado and a dome that can be set up at a campsite.





Geodesic domes are made as close to portions of spheres as is practical. Two reasons are that the sphere encloses the greatest volume with the least surface, and that it is the strongest shape.

A standard type of dome is based on a solid called an *icosahedron*. Make an icosahedron model using cardboard or straws and yarn from a pattern of 20 equilateral triangles.

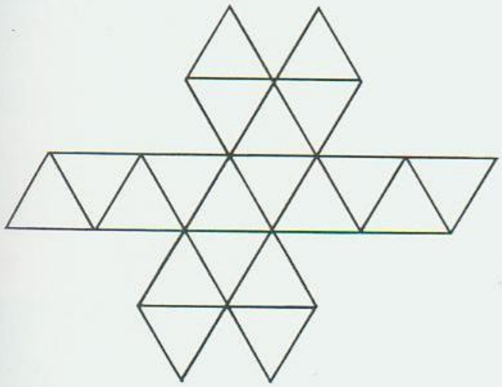
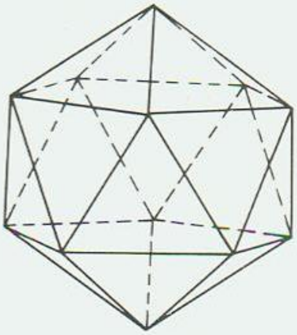


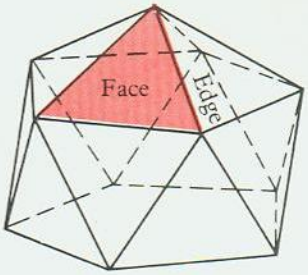
Fig. 1 Pattern for an icosahedron



icosahedron

If the bottom five triangles are removed a dome-like structure results.

Fig. 2



In an actual dome structure each of the equilateral triangles in Fig. 2 is divided into smaller triangles as shown in Fig. 3. In order for the shape of the dome to be closer to a sphere, pieces used to form the triangles, called *struts*, are made in unequal lengths. Type *C* struts are slightly longer than type *B* struts, and type *B* struts are slightly longer than type *A* struts.

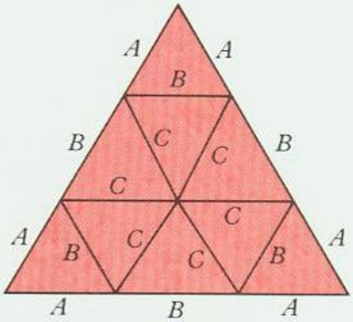


Fig. 3 Face of an icosahedron, divided into 9 triangles

1. How many struts of type *A* will be needed to complete the dome in Fig. 2? type *B*? type *C*?
2. In this dome, how many triangles of sides *A*, *A*, *B* are there? of sides *B*, *C*, *C*?